AERATED JETS AND PRESSURE FLUCTUATION IN PLUNGE POOLS

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ABSTRACT

In this paper are presented the principal results of a theoretical analysis and revaluation of experimental data corresponding to different investigation about mean and fluctuating dynamic pressure in plunge pools. These results are discussed and compared with the case of circular jets and it is demonstrated that knowledge of the falling jet process is of crucial importance for the characterization of the downstream physical phenomena.

Thus, observations and formulae are proposed in the following subjects: initial jet turbulence intensity $T_u$, jet break-up length $L_b$, impingement jet thickness $B_j$, mean dynamic pressure coefficient $C_p$ and fluctuating dynamic pressure coefficient $C'_p$. Some examples are provided to demonstrate how the findings may be used in practice and also like a verification of the proposed methodology.

Keywords: dams, spillways, weirs, plunge pool, energy dissipation, pressure fluctuation

1. INTRODUCTION

There are two types of plunge pools energy dissipators. The first type is a circular jet, an example of which is Morrow Point Dam in the USA. Figure 1 shows a lateral and frontal view of the jets.

Figure 1 Morrow Point Dam (USA).

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The second type is the rectangular jet or nappe flow, for example the dams of Eume, Baserca and El Atazar in Spain (See Fig. 2, Fig. 3 and Fig. 4) and Gebidem Dam in Switzerland (see Fig. 5). The selection of the plunge pool depth is usually a technical and economic decision between a deep pool, which needs no lining, or a shallow pool, which needs a liner. Therefore, a designer needs to know the magnitude, frequency and extent of the dynamic pressure fluctuations on the pool floor as a function of the jet or nappe characteristics.

Figure 2 Eume Dam. Frontal and lateral views (Spain).

Figure 3 El Atazar Dam (Spain).

Figure 4 Baserca Dam (Spain).

Figure 5 Gebidem Dam (Switzerland).
The characterization of pressures in plunge pools has been obtained using different scale models. From the early works of Moore (1943), Lencastre (1961), Cola (1965), Beltaos (1976), Xu-Do-Ming et al. (1983), Lemos (1984), Cui Guang Tao et al. (1985), Ervine and Falvey (1987), Withers (1991), Ervine et al. (1997) and most recently Bollaert (2002).

In Spain these line of research has been undertaken at Technical University of Cataluña by Castillo (1989), Armengou (1991), Castillo et al. (1991), Puertas (1994), Castillo (1998), Castillo et al. (1999) and Castillo et al. (2004).

The principal mechanism of energy dissipation are the spreading of the plunging jet (aeration and atomization in the air), air entrainment by the entering jet and diffusion in the pool and finally, the impact with the pool base (see Fig. 5).

For design considerations we define both the issuance conditions and the impingement conditions. The issuance conditions, located at the exit of the spillway structure are define by the mean velocity \( V_i = \sqrt{2gh_0} \), where \( h_0 \) is approximately equal to two times the energy head \( h \).

The principal impingement conditions situated at entrance to the pool are the mean velocity, \( V_j \), and the impingement jet thickness, \( B_j = B_g + \xi \), where, \( B_g \), is the thickness by gravity conditions and \( \xi \), is the jet lateral spread distance by turbulence effect.

An important parameter to define here is the jet break-up length, \( L_b \), beyond this distance the jet is completely developed it no longer contains a core but essentially consist of blobs of water that disintegrate into a finer and finer drops.

Individual blobs and drops of water slow down due to air drag and eventually reach terminal velocity. The latter occurs when drag introduced by the air equals the weight of individual water globules or drops. Such interaction limits the erosive capacity of a fully developed jet (Annandale, 2006).
Once the jet hits the pool surface the air is entrained by the entering jet, the diffusion begins and the solid part of the jet is completely disintegrated by a depth of approximately four times the impingement thickness, $B_j$ (established flow).

Figure 6 Schematic of plunging jet instability and break-up. Adapted from Ervine et al. (1997).

The disintegration conditions of circular jets have been thoroughly mainly by Ervine et al. (1997), who produced different formulae.

However, in the case of rectangular jets or nappe flow have not been studied in any grade depth. The only expression that we know was proposed by Horeni (1956) for the jet break-up length.

2. **ESTIMATION OF THE INITIAL TURBULENCE INTENSITY IN THE NAPPE FLOW CASE**

Here, we are going to concentrate on the nappe flow case. In order to estimate the initial jet turbulence intensity, we will use as a starting point, the experimental equation of the break-up length for circular jet, established by Ervine et al. (1997):

$$\frac{L_{ij}}{D_j F_j^2} = \frac{1.05}{C^{0.82}}$$  \hspace{1cm} (1)
where $D_i$ and $F_i$ are the diameter and Froude number at issuance conditions, respectively, $C$ is the turbulence parameter defined as

$$C = 1.14T_u F_i^2$$ (2)

$T_u$ is the initial turbulence intensity. So the jet break-up length for nappe flow case would obey the following general form:

$$\frac{L_b}{B_i F_i^2} = \frac{K}{C^{0.82}}$$ (3)

where $B_i$ is the jet thickness at issuance conditions. If the Horeni's expression for rectangular jet

$$L_b = 6q^{0.32}$$ (4)

is transformed into a function of the general jet break-up length, we have

$$\frac{L_b}{B_i F_i^2} = \frac{6q^{0.32}}{B_i F_i^2} \left(\frac{1.14T_u F_i^2}{(1.14T_u F_i^2)^{0.82}}\right)$$ (5)

We observe that the $K$ coefficient is

$$K = \frac{6q^{0.32}}{B_i F_i^2} \left(\frac{1.14T_u F_i^2}{(1.14T_u F_i^2)^{0.82}}\right)$$ (6)

Moreover, the jet velocity when leaves the weir spillway in arch dam (velocity at issuance) is $V_i = \sqrt{2gh_0}$, where $h_0 \approx 2h$. The energy head in function of specific flow is

$$h = \left(\frac{q}{C_d}\right)^{2/3}$$ (7)

If we replace in (6) and make the respective manipulations, then we could obtain an estimator of the Turbulence Intensity in function of specific flow, for rectangular jet or nappe flow case

$$T_u^* = \frac{q^{0.43}}{IC}$$ (8)

where $IC$ represents the initial conditions of flow at issuance; so that

$$IC = \frac{14.95g^{0.50}}{K^{1.22}C_d^{0.19}}$$ (9)

The discharge coefficient is $C_d \approx 2.1$ in hydrodynamic spillway case (Units International System).

Experimental data on break-up lengths (for example Horeni, 1956) relate to horizontally issuing jets. It was supposed that gravity would not effect the jet break-up length considerably, and this has been supported by small scale jet evidence. However, some theoretical and experimental data reveals the effect of gravity in the case of vertical jet and, it was found that the break-up length of a contracting
jet is larger than a horizontally issuing jet (Takahashi and Kimura, 1972; Withers, 1991). So, is proposed a new estimator of the jet break-up length and must be applied from 0.25 m$^2$/s until the prototype values because are obtained more realistic results. For flows smaller than 0.25 m$^2$/s (laboratory tests values), the Horeni’s formula seems to be correct.

For circular jet, $K \approx 1.05$. However, the break-up length at circular jet is very much longer than at rectangular jet (circular jet is very much compact), for example for $T_u=1\%$ and $q = 10$ m$^2$/s, the value of $L_b \approx 120$ m in circular jet case, and possibly only $L_b \approx 45$ m in rectangular jet (see Fig. 7), so that $K$ can vary depending on the particular case.

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![Figure 7 Jet break-up length for rectangular and circular jet.](image)

We can observe in Fig. 7 that the values from the new estimator ($T_u =1.2\%$) are very similar for the case of circular jet ($T_u =3\%$). So, for equals specific flows, the circular jet is much more compact that the rectangular jet. The new estimator proposed (see justification in the next section) to calculate the jet break-up length in the rectangular jet or nappe flow case is:

$$\frac{L_b}{B_i F_i^2} = \frac{0.85}{(1.07 T_u F_i^2)^{0.82}}$$

(10)

2.1 Estimation of the impingement jet thickness

The impingement jet thickness is

$$B_j = B_g + B_s = B_g + 2\xi$$

(11)
where $B_g$ is the thickness by gravitational considerations, $B_s$ is the thickness by lateral spread and $\xi$ is the lateral spread distance of turbulent jet in the atmosphere. Following Ervine et al. (1997)

$$\xi = k v t = k \left( \frac{v'}{V_i} \right) \frac{V_i - V_j}{g} \tag{12}$$

where we define a turbulence parameter $\phi = k \left( \frac{v'}{V_i} \right) = k T_u'$; $t$ is the time for the jet to fall any distance; $v'$ is the streamwise turbulent component; $V_i$ and $V_j$ are the mean jet velocity at issuance and impingement conditions, respectively.

If we replace the mean velocities in (12), then

$$\xi = 2 \phi \sqrt{h_0} \left[ \sqrt{H} - \sqrt{h_0} \right] \tag{13}$$

so, the impingement thickness for rectangular jet or nappe flow case is:

$$B_j = \frac{q}{2gH} + 4 \phi \sqrt{h_0} \left[ \sqrt{H} - \sqrt{h_0} \right] \tag{14}$$

where $H$ is the water level difference between upstream and downstream of the structure and $h_0$ is equal to two times the energy head at the spillway, $h_0 \approx 2h$ (see Fig. 5).

Figure 8 Turbulence velocities in circular jet.

Ervine and Falvey (1987) postulated that the transverse turbulent velocities are $u' = w'$ and $u' = 0.38 v'$; so that the root-mean-square of the streamwise turbulent component is

$$v' = \sqrt{u'^2 + v'^2 + w'^2} = 1.14 v' \tag{15.a}$$

the lateral spread is

$$\xi = k T_u' \sqrt{v' t} = 1.14 v' t \tag{15.b}$$

In rectangular jet or nappe flow case, we estimate that in the central line of the jet $u' << w'$ and if we accept that $w' = 0.38 v'$, then the root-mean-square of the streamwise turbulent component is

$$v' = \sqrt{(0.38 v')^2 + v'^2} = 1.07 v' \tag{16.a}$$
then the lateral spread it would be
\[ \xi = kT_u V_t = 1.07v't \]  \hspace{1cm} (16.b)

So the estimator of the turbulent parameter at the jet impingement conditions for nappe flow is
\[ \varphi = 1.07T_u^* \]  \hspace{1cm} (17)

Figure 10 shows a first verification of the method that was obtained in a small model, Castillo (1989). It was carried out a sensitivity analysis for \( K \) values between 0.44 and 0.95. From the results obtained we can conclude that for rectangular jet or nappe flow case this values could be with more probability between \( 0.60 \leq K \leq 0.85 \). We have established \( K \approx 0.85 \), like a value of the safe side, although it will be necessary to obtain further information in models and prototypes.
3. MEAN DYNAMIC PRESSURE COEFFICIENT, $C_p$

For circular jets there is an exhaustive analysis of the mean dynamic pressure coefficients, obtained in model with velocities greater than 20 m/s and turbulence intensity until 5 % (see Ervine et al. 1997, Bollaert, 2002). However, these coefficients correspond in general to jet break-up length $H/L_b \leq 0.50$.

For nappe flow or rectangular jet, if $L_b$ is estimated with Horení´s formula, the laboratory data that we dispose cover the following range:

- Castillo’s (1989) data cover a range of $0.50 \leq H/L_b \leq 0.90$. The data correspond to different falling heights $H$ between 1.60 m to 1.76 m, seven water cushion heights $Y (0–0.04–0.08–0.12–0.16–0.20–0.25 \text{ m})$ and three specific flows $q (0.0125–0.0250–0.050 \text{ m}^2/\text{s}) (3–6–8 \text{ l/s})$.

- Puertas’ (1994) data cover a range of $0.50 \leq H/L_b \leq 2.70$. Four falling heights with $H (1.85–2.88–4.43–5.45 \text{ m})$, ten water cushions heights $Y (0.08–0.16–0.24–0.32–0.40–0.48–0.56–0.80 \text{ m})$ and a range of the specific flows $q (0.026 \text{ to } 0.15 \text{ m}^2/\text{s}) (31.2 \text{ to } 180 \text{ l/s})$.

![Figure 11 Sketch of the experimental structure (Puertas and Dolz, 2002).](image)

Castillo’s and Puertas’s instantaneous pressures were registered on the bottom of the pool, by means of piezoresistive pressure transducers. It was obtained around of 200 registers, each one of 2400 points with a rate of data acquisition of 20 points per second.

Puertas (1994) made a multivariant treatment of the most outstanding non-dimensional variables.

The expression proposed is:

$$\frac{\Delta p_{\text{max}}}{H \gamma} = \frac{3.88q}{Y \sqrt{2gH}}$$  \hspace{1cm} (18)

The expression is valid whenever an effective cushion $Y_e$ is guaranteed.
\[ Y_e > \left[ \frac{0.113}{\sqrt{2g}} Hq \right]^{2/5} \]  

However, Puertas´s formulation is the product of a global treatment of data and for this reason underestimates the \( C_p \) coefficient (see examples in the last section heading). Figure 12 shows the variation of the mean dynamic pressure coefficient, \( C_p \), in function of the parameter \( Y/B_j \), corresponding to the experimental data from Castillo (1989) and Puertas (1994). We can observe that there are a spreading of data when it is not considered the parameter \( H/L_b \). 

Figure 12 Variation of the mean dynamic pressure coefficient in function of \( Y/B_j \).

Castillo (1998) carried out a new analysis with Puertas´s and Castillo´s data and proposed formulations of \( C_p=f(Y/B_j, H/L_b) \). In Figure 13 these results are presented and for comparison, the principals results for circular jets (aerated and no aerated) from another authors, are also shown.
We can see that for the case of $H/L_b \leq 0.5$ we obtain a single curve. However, for $H/L_b > 0.5$ it is obtained a family of curves in function of this parameter.

We can also see from this figure that the disintegration of the solid part of the jet, occurs at a depth of approximately four times of the impingement jet thickness ($Y < 4B_j$, zone of flow establishment). In this range is valid the next potential regression curve $C_p = f(H/L_b)$ and whose regression coefficient $R^2 = 0.99$ is excellent:

$$C_p = 0.36(H/L_b)^{-1.04} \quad (20.a)$$

The curve of the energy dissipation both in the air and by the degree of air entrainment into the plunge pool, $DE_{air}$, is obtained as the difference between the one value minus the $C_p$ value:

$$DE_{air} = 1 - 0.36(H/L_b)^{-1.04} \quad (20.b)$$
For $H/L_{b} > 0.5$, the general formulation to obtain the mean dynamic pressure coefficient for aerated rectangular jet or nappe flow case, follows a exponential law:

$$C_{p} = \frac{H_{m} - Y}{V_{j}^{2} / 2g} = ae^{-b(Y/B_{j})}$$

where $H_{m}$ and $Y$ are the mean head and depth at plunge pool; $V_{j}$ and $B_{j}$ are the velocity and thickness of the impingement jet. The parameters are shown in Table 1, the minimum standard deviation and regression coefficient obtained for different curves fitting was $R=0.90$ and $R^{2}=0.81$, respectively.

<table>
<thead>
<tr>
<th>$H/L_{b}$</th>
<th>$a$</th>
<th>$b$</th>
<th>$C_{p}$ ($Y/B_{j} &lt;= 4$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt; 0.5</td>
<td>0.98</td>
<td>0.070</td>
<td>0.78</td>
</tr>
<tr>
<td>0.5-0.6</td>
<td>0.92</td>
<td>0.079</td>
<td>0.69</td>
</tr>
<tr>
<td>0.6-0.8</td>
<td>0.65</td>
<td>0.067</td>
<td>0.50</td>
</tr>
<tr>
<td>1.0-1.3</td>
<td>0.65</td>
<td>0.174</td>
<td>0.32</td>
</tr>
<tr>
<td>1.5-1.9</td>
<td>0.55</td>
<td>0.225</td>
<td>0.22</td>
</tr>
<tr>
<td>2.0-2.3</td>
<td>0.50</td>
<td>0.250</td>
<td>0.18</td>
</tr>
<tr>
<td>&gt; 2.3</td>
<td>0.50</td>
<td>0.400</td>
<td>0.10</td>
</tr>
</tbody>
</table>

Table 1 Parameters of the exponential law of the mean dynamic pressure coefficients in function of the different jet break-up length.
4. **FLUCTUATING DYNAMIC PRESSURE COEFFICIENT, \( C' \)**

For circular jet case, the root-mean-square value of the pressure fluctuation depends on both the \( Y/B_j \) ratio and the initial turbulence intensity of the jet \( T_u \). Bollaert’s (2002) and Bollaert and Schleiss (2003) data have been obtained with velocities higher than 20 m/s and for this reason, they affirm that the results are exempt of scale effects and, thus, representative for prototype jets.

Figure 15 shows the results from Castillo (1989) and Puertas (1994) data. We can observe that there is not a grouping clear with respect to the ratio of fall height per jet break-up length, \( H/L_b \). It was analyzed and detected fifteen outliers that disturb the general tendency and if are discounted, then we are able to grouping it in three principal zones, so \( H/L_b \leq 1.4, 1.4 < H/L_b \leq 2 \) and \( H/L_b > 2 \).

![Figure 15](image)

**Figure 15** Fluctuating dynamic pressure coefficient for rectangular jet. From Castillo (1989) and Puertas (1994) data.

Figure 16 shows the results from Bollaert (2002) for different turbulence intensity, \( T_u \) (circular jet) and Castillo (2006), in where is considered the parameter \( H/L_B \) (rectangular jet).

Although in the aerated rectangular jet or nappe flow case, the velocities in the tests were only reached up to 10 m/s; the maximum coefficient is \( C' \approx 0.31 \) (\( H/L_B < 1.4 \)) and is in good accordance with the best fit of Bollaert (2002) for 3 % < \( T_u \) < 5 % but corresponding it a value of \( Y/B_j \approx 5 \).
For $1.4 < H/L_B \leq 2$, is obtained a $C_p' \approx 0.23$ and with a value of $Y/B_j \approx 5$. This value is also in good accordance with the best fit of Bollaert for $1 \% < T_u < 3 \%$. Finally for $H/L_B > 2$, the value of $C_p' \approx 0.12$ and with $Y/B_j \approx 5$. These values constitute a new verification of the method and are in accordance with these structures type.

The best fit obtained to quantify the fluctuating dynamic pressure coefficient as a function of the parameters $Y/B_j$ and $H/L_b$ is obtained with two fit types:

**Polynomial fit:**

$$C_p' = a(Y / B_j)^3 + b(Y / B_j)^2 + c(Y / B_j) + d$$  \hspace{1cm} (22.a)

for $Y/B_j < 14$.

**Potential fit:**

$$C_p' = a(Y / B_j)^b$$  \hspace{1cm} (22.b)

for $Y/B_j \geq 14$.

The relationships between the jet fall height to break-up length, $H/L_b$ and the dimensionless coefficients $a$, $b$, $c$ and $d$, are presented in Table 2.
### Table 2  Coefficient values for calculating the fluctuating dynamic pressure coefficient.

<table>
<thead>
<tr>
<th>$H/L_b$</th>
<th>$a$</th>
<th>$b$</th>
<th>$c$</th>
<th>$d$</th>
<th>Type of the jet</th>
</tr>
</thead>
<tbody>
<tr>
<td>≤ 1.4</td>
<td>0.0003</td>
<td>-0.0104</td>
<td>0.0900</td>
<td>0.083</td>
<td>Compact–Developed-Disintegrated</td>
</tr>
<tr>
<td>1.5 – 2</td>
<td>0.0003</td>
<td>-0.0094</td>
<td>0.0745</td>
<td>0.05</td>
<td>Developed-Disintegrated</td>
</tr>
<tr>
<td>&gt; 2</td>
<td>0.0002</td>
<td>-0.0061</td>
<td>0.0475</td>
<td>0.01</td>
<td>Developed-Disintegrated</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$H/L_b$</th>
<th>$a$</th>
<th>$b$</th>
<th>Type of the jet</th>
</tr>
</thead>
<tbody>
<tr>
<td>≤ 1.4</td>
<td>5.30</td>
<td>-1.405</td>
<td>Compact–Developed-Disintegrated</td>
</tr>
<tr>
<td>1.5 – 1.4</td>
<td>3.14</td>
<td>-1.422</td>
<td>Developed-Disintegrated</td>
</tr>
<tr>
<td>&gt; 2</td>
<td>1.50</td>
<td>-1.500</td>
<td>Developed-Disintegrated</td>
</tr>
</tbody>
</table>

#### 5. EXAMPLES

The examples chosen are the Arches Dams of the La Llosa del Cavall, Los Angeles and Susqueda (Spain), Morrow Point Dam (USA) and Kariba Dam (border between Zambia and Zimbabwe).

<table>
<thead>
<tr>
<th>Dam</th>
<th>$q$ (m$^2$/s)</th>
<th>$H$ (m)</th>
<th>$h_0$ (m)</th>
<th>$V_i$ (m/s)</th>
<th>$B_i$ (m)</th>
<th>$F_i$</th>
<th>$Y$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>La Llosa del Cavall</td>
<td>9.85</td>
<td>97.15</td>
<td>7.12</td>
<td>11.82</td>
<td>0.83</td>
<td>4.14</td>
<td>15.60</td>
</tr>
<tr>
<td>Los Angeles</td>
<td>8.57</td>
<td>29.55</td>
<td>5.10</td>
<td>10.00</td>
<td>0.86</td>
<td>3.44</td>
<td>10</td>
</tr>
<tr>
<td>Susqueda</td>
<td>23.20</td>
<td>105</td>
<td>7.66</td>
<td>12.26</td>
<td>1.89</td>
<td>2.85</td>
<td>13.50</td>
</tr>
<tr>
<td>Morrow Point (circular jet)</td>
<td>58.94</td>
<td>110</td>
<td>-</td>
<td>12.50</td>
<td>$D_i = 5.36$</td>
<td>1.72</td>
<td>20</td>
</tr>
<tr>
<td>Morrow Point (circular jet)</td>
<td>159.10</td>
<td>59</td>
<td>-</td>
<td>17.48</td>
<td>$D_i = 10.1$</td>
<td>1.76</td>
<td>20</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Kariba (circular jet)</th>
<th></th>
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</table>

Table 3  Principal data of the dams.
5.1 La Llosa del Cavall Dam

Hydraulic model studies by Dolz et al. (1990) have revealed mean heads on the plunge pool floor up to 33-45 m, including the plunge pool depth of 15.60 m. The mean dynamic head, alone, reached 18-30 m, giving $C_p$ values of the order of 0.19 to 0.30. These values obtained in the model (without aeration and with action of the superficial tension) are greater than the values that will be obtained in the prototype.

From Castillo’s formulae:
* Assume Turbulence intensity when the flow pass on spillway (critical conditions) $T_u = 0.012$
* From Equation (10), the estimate break-up length is $L_b = 41.99$ m
* The relation $H/L_b = 2.31$
* From Equations (8) an (9), the turbulence intensity at issuance conditions $T_u^* = 0.054$
* From Equation (14), the impingement jet thickness is $B_e = 4.65$ m
* The relation $Y/B_j = 3.35$. Because $Y/B_j < 4$, then the energy dissipation in the water cushion is null
* From Equation (20.a) or Fig. 13, the mean dynamic pressure coefficient is $C_p = 0.15$
* From Equation (22a) or Fig. 16, the fluctuating mean dynamic pressure coefficient is $C'_p = 0.11$.

From Puertas’s formula:
* From Equation (18), the mean dynamic pressure coefficient is $C_p = 0.06$. This is a very low value

5.2 Los Angeles Dam

From Castillo’s formulae:
* Assume Turbulence intensity when the flow pass on spillway (critical conditions) $T_u = 0.012$
* From Equation (10), the estimate break-up length is $L_b = 40.43$ m
* The relation $H/L_b = 0.73$
* From Equations (8) an (9), the turbulence intensity at issuance conditions $T_u^* = 0.051$
* From Equation (14), the impingement jet thickness is $B_e = 1.92$ m
* The relation $Y/B_j = 5.22$. Because $Y/B_j > 4$, then there is energy dissipation in the water cushion
* From Equation (21) or Fig. 13, the mean dynamic pressure coefficient is $C_p = 0.46$
* From Equation (22a) or Fig. 16, the fluctuating mean dynamic pressure coefficient is $C'_p = 0.25$.

From Puertas’s formula:
* From Equation (18), the mean dynamic pressure coefficient is $C_p = 0.14$. This is a very low value

5.3 Susqueda Dam

From Castillo’s formulae:
* Assume Turbulence intensity when the flow pass on spillway (critical conditions) $T_u = 0.012$
* From Equation (10), the estimate break-up length is $L_b = 83.35$ m
* The relation $H/L_b = 1.26$
* From Equations (8) an (9), the turbulence intensity at issuance conditions $T_u^* = 0.078$
* From Equation (14), the impingement jet thickness is $B_j = 7.42$ m
* The relation $Y/B_j = 1.82$. Because $Y/B_j < 4$, then the energy dissipation in the water cushion is null
* From Equation (20.a) or Fig. 13, the mean dynamic pressure coefficient is $C_p = 0.28$
* From Equation (22a) or Fig. 16, the fluctuating mean dynamic pressure coefficient is $C'_p = 0.19$. 
From Puertas’s formula:
* From Equation (18), the mean dynamic pressure coefficient is $C_p = 0.15$. This is a very low value

5.4 Morrow Point Dam (Circular jet case)

In this example is necessary combine two methods. The first to calculate the jet break-up length and the impingement jet thickness for circular jet (Ervine et al., 1997) and second to calculate the mean dynamic pressure coefficient (Castillo, 1998 or Puertas, 1994).

Hydraulic model studies by King et al. (1966) have revealed mean heads on the plunge pool floor up to 60-65 m, including the plunge pool depth of 20 m. The mean dynamic head, alone, reached 40-45 m, giving $C_p$ values of the order of 0.30 to 0.4.

From Castillo’s formulae:
* Assume Turbulence intensity $T_u = 0.05$ (This is estimated from photographs of jet spread in the atmosphere, Ervine et al., 1997)
* From Equation (2.12) (Ervine et al., 1997):

$$C^2 = 1 \left( (2L_b/(D_0F_0^2)) + 1 \right) \left( (2L_b/(D_0F_0^2)) + 1 \right)$$

the theoretical jet break-up length for circular jet is $L_b = 120$ m
* The relation $H/L_b = 0.92$
* The impingement jet thickness is $B_e = 8$ m (Ervine et al., 1997)
* The relation $Y/D_j = 2.5$. Because $Y/B_j < 4$, then the energy dissipation in the water cushion is null
* From Equation (20.a) or Fig. 13, the mean dynamic pressure coefficient is $C_p = 0.39$
* From Equation (22a) or Fig. 16, the fluctuating mean dynamic pressure coefficient is $C_p' = 0.20$
* From Ervine et al. (1997), the fluctuating mean dynamic pressure coefficient is $C_p' = 0.24$. From Fig. 16 (Bollaert, 2002. Circular jet $T_u = 5\%$), $C_p' = 0.23$

From Puertas´s formula:
* From Equation (18), the mean dynamic pressure coefficient is $C_p = 0.25$. This is a very low value

5.5 Kariba Dam (Circular jet case)

In this example for $Y=20$ m the mean dynamic pressure coefficient can be only applied the method of Ervine, et al., 1997 ($Y/D_j=1.47$ then $C_p=0.86$).

The dam is founded on very hard to extremely hard gneiss rock, which has experienced significant scour over the years. The current depth of the scour hole is approximately 80 m.
* Assume Turbulence intensity $T_u = 0.08$
* From Equation proposed by Ervine et al., (1997):

$$C^2 = 1 \left( (2L_b/(D_0F_0^2)) + 1 \right) \left( (2L_b/(D_0F_0^2)) + 1 \right)$$

the theoretical jet break-up length for circular jet is $L_b = 255$ m
* The relation $H/L_b = 0.23$
* The impingement jet thickness is $D_j = D_i \sqrt{V_i/V_j} + 2 \varepsilon = 13.56$ m

$$V_j = \sqrt{V_i^2 + 2gH} = 38.25 \text{ m/s}$$

$$\varepsilon = ((1.14T_uV_i^2) / g) \sqrt{2L_b/(D_iF_i^2)} + 1 - 1 = 3.364 \text{ m}$$
* The relation $Y/D_j = 20/13.56 = 1.47$. Because $Y/D_j < 4$, then the energy dissipation in the water
The current depth of the scour hole is 80 m, then $Y = 100$ m and the relation $Y/D_j = 7.37$. Because $Y/D_j > 4$, then there is energy dissipation in the water cushion and the mean dynamic pressure coefficient is 

$$C_p = 38.41(1 - C_j)(D_j/Y)^2 = 0.39$$

The initial air jet concentration: $C_j = \beta_j/(1 + \beta_j) = 0.45$

The air-water ratio, due to a compact jet entering in pool:

$$\beta_j = Q_a/Q_w = K_1[(1 - (V_m/V_j))\sqrt{H/D_j} = 0.813$$

where $Q_a$ and $Q_w$ are the air entrained discharge rate and the water flow rate, respectively, and $V_m$ is the minimum velocity required to entrain air by a plunging jet (about 1 m/s). The value $K_1$ varies from 0.2 for smooth turbulent circular jets to around 0.4 for very rough turbulent jets (Ervine et al., 1997).

* From Equation (22a) or Fig. 16, the fluctuating mean dynamic pressure coefficient is $C'_p = 0.30$. From Ervine et al. (1997), the fluctuating mean dynamic pressure coefficient is $C'_p = 0.20$. From Fig. 16 (Bollaert, 2002. Circular jet $T_u = 8\%$), $C'_p = 0.34$

We can observe that the original mean dynamic pressure coefficient $C_p=0.86$, has going to producing the scour pool development, reducing the mean dynamic coefficient progressively until to reach the equilibrium depth of $Y=100$ m and a value of $C_p=0.39$. However, the fluctuating mean dynamic pressure coefficient increasing from Ervine ($C'_p=0.05$ to 0.20) and from Bollaert ($C'_p=0.24$ to 0.34). This would explicate why the scour could continue yet.

<table>
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<th>Dam</th>
<th>$q$ (m$^2$/s)</th>
<th>$H$ (m)</th>
<th>$Y$ (m)</th>
<th>$H/L_b$ (m/s)</th>
<th>$Y/B_j$ (m)</th>
<th>$C_p / (C'_p)$</th>
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* This values are calculated with Bollaert (2002) formulae for circular jets.

Table 4  Resume of the principal results.
Figure 17 shows the representation of the $C_p$ coefficients both the registered (in hydraulics models) as the calculated (Castillo, 2006 and Puertas, 1994). We can observe that the registered coefficients are in almost all the cases moderately greater than the calculated coefficients from Castillo’s formulae but very much greater than the calculated from Puertas’s formula. In any way, the registered coefficients must be greater than calculated since at reduced models with Froude similarity, the effects of the superficial tension are not considered neither the aeration phenomena.

![Figure 17 Mean dynamic pressure coefficient, $C_p$, both registered (hydraulic models) and calculated (Castillo, 2006 and Puertas, 1994).](image)

For the rectangular jet case, the mean dynamic pressure coefficients calculated with Castillo’s formulae could be considered as the values more closed to the prototype values and we can observe that in general this coefficients follows correctly the variations of the main parameters, $H/L_b$ and $Y/B_j$. For the circular jet case, the Ervine’s formulae is the that produce the correct results. We can observe that for Kariba Dam ($Y/B_j=1.47$), the value obtained with Puertas’s formula exceed the limit value of 0.86 corresponding to circular aerated jet. However, for Kariba Dam ($Y/B_j=7.37$) the value from Puertas’s formula is excessively small (0.01).

Figure 18 indicates the variation of the mean dynamic pressure coefficient, $C_p$, in function of rate fall height per jet break-up length, $H/L_b$. We can observe that when the values of the parameter $H/L_b$ increases, the values of the registered and measured coefficients, $C_p$, diminish progressively, coming to converge to the value of $C_p \approx 0.07$ when $H/L_b= 4.5$.

In Fig. 18 is also indicated the sum of the mean and fluctuating dynamic pressure coefficient ($C_p + C_p'$. This is the value that must be used for the concrete line slab design. We can see that in almost all cases these values are inside of the range of the registered values in scale models, or are slightly greater. However, for Morrow Point and Kariba, whose typologies correspond to circular jet, there is a logical deviation because the Castillo’s formulae correspond to rectangular jet.
Figure 19 shows the variation of the mean dynamic pressure coefficient, $C_p$, in function of the specific flow, $q$. When the values of the specific flow increase then the values of the registered and measured coefficients, $C_p$, increase progressively too. The coefficients ($C_p + C'_p$) are also indicated, and we can see that are inside of the registered values in scale models or are slightly greater but always of the safe side (exception is Morrow Point, circular jet typology).

Figure 18 Variation of the mean dynamic pressure coefficient, $C_p$, and mean and fluctuating pressure coefficients, ($C_p + C'_p$), in function of rate of fall height per jet break-up length, $H/L_b$.

Figure 19 Variation of the mean dynamic pressure coefficient, $C_p$, and mean and fluctuating pressure coefficients, ($C_p + C'_p$), in function of the specific flow, $q$. 
5. LIST OF SYMBOLS

\( B_g \)  jet thickness by gravitational consideration  
\( B_j \) minimum thickness of rectangular jet or nappe flow at entry point  
\( B_s \) jet thickness by lateral spread  
\( D E_{air} \) Energy dissipation in the air and by the degree of air entrainment into the plunge pool  
\( H \) falling height  
\( H_m \) mean head in plunge pool  
\( I C \) initial conditions at issuance  
\( K \) proportional coefficient for break-up length of rectangular jet or nappe flow  
\( k \) proportional coefficient for lateral spread distance of turbulent jet  
\( L_b \) jet break-up length  
\( q \) discharge per unit width or specific flow of rectangular jet or nappe flow  
\( T_u \) turbulence intensity when the flow pass on the spillway (critical conditions)  
\( T_{u^*} \) turbulence intensity at issuance conditions  
\( u', w' \) transverse turbulent velocities  
\( v' \) streamwise turbulent component  
\( \overline{v'} \) root-mean-square of streamwise turbulent component  
\( V_i \) mean velocity at issuance condition  
\( V_j \) mean velocity at impingement condition  
\( Y \) water cushion height  
\( Y_e \) effective water cushion  
\( \Delta p \) mean dynamic pressure  
\( \gamma \) water specific weight  
\( \rho \) water density  
\( \phi \) turbulence parameter at impingement conditions  
\( \xi \) lateral spread distance  

5. CONCLUSIONS

Although analysis of rectangular jet or nappe flow for aerated jets is complicated, we have seen here a practical design methodology for this type of structures.
The Horeni’s formula to calculate the jet break-up length \( L_b \) in rectangular jet is valid in the range of the laboratory flows. To estimate \( L_b \) in prototypes \((q > 0.25 \, \text{m}^2/\text{s})\) it must be used the new estimator proposed, expression (10).

For the estimation of the jet thickness at impingement conditions must be used the expression (14) and this value is very important for the determination of the mean and fluctuating dynamic pressure coefficients and the jet footprint.
In the case of no aerated jet (circular and rectangular jet) or submerged jet, the mean dynamic pressure coefficient is greater than that of a comparable aerated jet. The \( C_p \) value is constant and equals to the fall total head \((C_p=1)\) if the jet core impacts on the pool bottom (flow establishment).

The zone of the flow establishment is greater than in the aerated jets cases. So, for no aerated circular jet \( Y/D_j \approx 6.2 \) and for no aerated rectangular jet \( Y/B_j \approx 7.8 \). However, in aerated jets both circular and rectangular jet, the zone flow establishment is \( Y/B_j \approx 4 \).
In aerated jets, when \( Y/B_j < 4 \), the mean dynamic head is \( C_p=0.86 \) (circular jet) and \( C_p=0.78 \) (rectangular jet). There are two reasons for that the values of the pressure coefficients only reaching these values. The first is the spreading of the plunging jet at the entry point of the pool (impingement conditions), thus, although most of the jets were relatively compact at entry, the
spreading of the jet is sufficient to reduce the pressure coefficient. The second reason is the air entrained by the entering jet in the pool.

The energy dissipation and the air entrained for rectangular jet case, can be calculated directly with the expression (20.b) and corresponds to the situation when there is not energy dissipation in the water cushion (flow establishment, $Y/B_j < 4$). This energy dissipation is more smaller than that of a comparable circular jet (aerated or no aerated jet).

In the case of established flow ($Y/B_j > 4$), the mean dynamic pressure coefficient can be obtained by means of the expression (21) and with the parameters of table 1 (in function of the $H/L_b$ parameter) or directly from Figure 13. The energy dissipation by diffusion in a circular jet is greater than the of a comparable rectangular jet.

The values of the fluctuating dynamic pressure coefficients of the rectangular jet have a great similitude with the circular jet ones. Figures 15 and 16 shows the fit curves of the $C'_p$ coefficient in function of $Y/B_j$ (or $Y/D_j$ in the case of circular jet) and for different relations $H/L_b$ (or $T_u$ in the case of circular jet). With the available data only it was possible contain three groups of $C'_p$ coefficient in function of $H/L_b$ parameter. The maximum values are obtained for $Y/B_j \approx 5$ and can be higher values as $C'_p=0.30$ for ($H/L_b \leq 1.4$), reducing it to $C'_p=0.22$ for ($1.4<H/L_b \leq 2$) and $C'_p=0.15$ for ($H/L_b>2$). This coefficients can be calculated by means of the expressions (22a) and (22b) and the parameters of table 2 (in function of $H/L_b$ parameter) or directly from figures 15 or 16.

From the results obtained by the application of the this methodology to the different plunge pool of arch dams and their contrast with the results obtained in the different hydraulic models, provided a fair amount of confidence in the proposed relations for calculate the jet break-up length, impingement jet thickness, energy dissipation in the air and entrained by the entering jet in the pool, mean and fluctuating dynamic pressure coefficients. It is necessary remember that the total dynamic pressure is the sum of the mean and fluctuating dynamic pressure.

A limitation of the described work is that both the disintegration length and turbulence intensity have been assumed and adapted from the results of others authors.

In order to improve the methodology, further measurements are required from models and prototypes, specially concern turbulence and aeration. At the Hydraulic Laboratory of the Technical University of Cartagena UPCT, a substructure is being constructed to make tests with some falling heights and flow ranges.

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Dr. Jerónimo Puertas (La Coruña University) and Dr. José Dolz (Technical University of Cataluña) provided a lot of experimental data on rectangular nappes.

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