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Parametrical analysis of the ultimate scour and mean dynamic pressures at plunge pools

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ABSTRACT: Studies to find out the actions at dam toe by impingement jet effect have been carried out using two different ways of research: the study of scour and the study of pressures. The instrumentation and measurement objectives have marked the difference between the two ways of research and "apparently" the type of formulation obtained. In this paper the general formulation of limit scour and pressure is shown, analyzing its correspondence and showing that it responds to a unique type of formulation. A practical parameter; which we call "incremental energy dissipation", is presented in order to estimate the necessary resistance which the rigid basin floor would have to resist by the power of water jet. Thus allows the analysis of the pressure fluctuation which the plate o rock floor of the basin would have to resist.

1 INTRODUCTION

The studies to find out the actions at the dam toe through the effect of jet impingement, were carried out following two different ways of research: the study of scour plunge pools and the study of instantaneous, mean and fluctuating pressures. The instrumentation and objective of the measurements marked the difference between these two approaches and as a consequence, "apparently" the type of formulation obtained. Thus, in the scour formulations, the main measurement is the depth and shape of the scour hole, while in the second approaches the main objective is the characterization of the pressure on the plunge pool floor.

The classic scour formulae constitute empirical correlations obtained in a model: Schoklitsch (1932), Veronese (1937), Muñoz (1964), Rajaratnam (1981), Rajaratnam et al. (1995), in a prototype Wu (1973) and in prototype-model: Mason et al. (1985), Lopardo et al. (1987), etc..

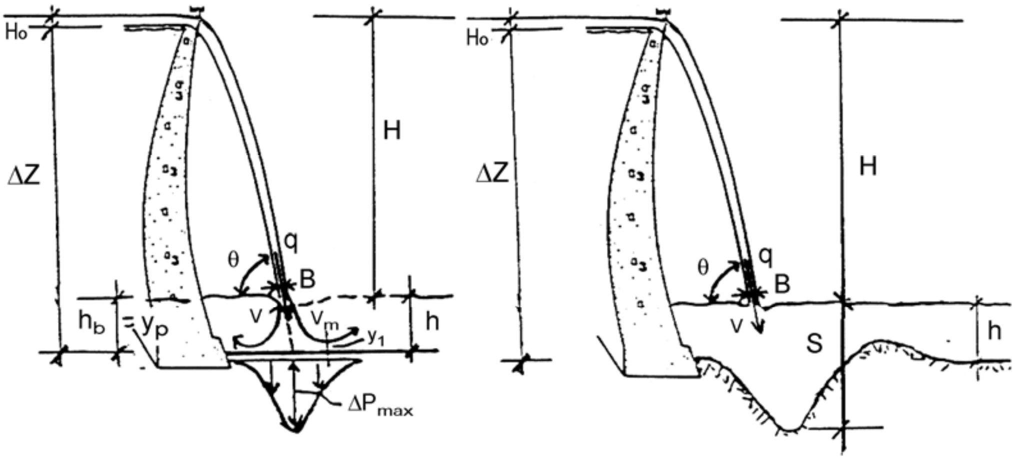
Ramírez et al. (1990) carried out a study of the joining and generalization of the different scour formulae by means of turbulent jet theory.

As far as the pressure characterization studies are concerned, the main empirical formulations have been determined exclusively in models, because of the complexity of installing instrumentation in prototypes. These formulations have evolved since the work of Moore (1943), Lencastre (1961), Cola (1965), Aki (1969), Hartung & Häusler (1973), Beltaos (1976) until the more recent work of Xu-Do-Ming et al. (1983), Lemos et al. (1984), Cui Guang Tao et al. (1985), Ervine & Falvey (1987), Withers (1991), Ervine et al. (1997) and the research program of the Hydraulics Laboratory of the Universidad Politécnica de Cataluña (UPC): Castillo (1989), (1990), Castillo et al. (1991), Armengou (1991), Puertas (1994), Castillo et al. (1996) and Castillo et al. (1999).

In this paper some of the general formulations of limit scour and pressures are presented, a correspondence is drawn up and they are transformed into a single type of formulation. In addition, a practical parameter; which we call "incremental energy dissipation" is presented in order to estimate the necessary resistance which the rigid basin floor would have to resist by the power of water jet. Thus allows the analysis of the different actions in the plunge pool.

2 ANALYSIS OF THE JET IMPINGEMENT

At overflow spillways the total energy is dissipated by the following mechanism (see Fig. 1): First by friction, spreading and air entrainment if jet reach velocities of 6 m/s (fall heights of over 1.83 m); and by the atomization of the water for jet velocities of 20-30 m/s (heights of 20 - 46 m). Second: if there is a rigid floor (Fig. 1a), by the combined effect of diffusion and impact and by internal friction through a submerged hydraulic jump in the plunge pool.
As regards a scour hole (Fig. 1b), the scour limit state is reached time-independent due to the total jet diffusion. The hole scour is the main mechanism, until the velocity of the jet diminishes to such a level, where the shear stress on the bed reach a critical value according to Shields (not necessarily numerically equal) [Ramírez et al. (1990)].



(a) (b)

Figure 1. Plunge pool at dam toe. (a): Concrete lined floor (rigid plate). (b): Scour hole.

That means that the scour will grow on until a water cushion is obtained which is sufficient for the turbulent jet to be diffused totally. In addition, the velocities and pressure fluctuations have diminished to a certain level, in such a way that they are unable to mobilize and extract materials from the bed. For this fully developed jet the following ratio could be applied:

$$S^* = K\left(\frac{V}{V^*_c}\right)^n = V^{+n} \tag{1}$$

where $S^* = S/(B\sin\theta)$; S = scour depth reached from the level of water to the deepest point of the pool; B = thickness of the jet impingement; θ = angle of the jet impingement; V = the velocity of the jet impingement; V^*_c = the critical friction velocity (shear stress threshold) so that the movement of particles is produced in the plunge pool; K and n are coefficients which depend on the shape of the jet and which theoretically have a value for a plane jet at K = 2.7 and n = 2 and for a circular jet at K = 6.4 y n = 1 [Tennekes & Lumley (1972)].

2.1 Theoretical discussion and joining of the limit scour formulae

Most scour formulae obey to the following common form:

$$S = K\frac{H^{\alpha} q^{\beta} h^{\delta}}{D_i^{\gamma}} \tag{2}$$

where H = total height up to downstream level; h = depth of initial water cushion; D_i = representative diameter of the particles which remain in the plunge pool and K, α, β, γ and δ are coefficients which the different authors have obtained experimentally.

Ramírez et al. (1990) obtained from the vertical free-fall configuration and in conjunction with the submerged turbulent jet theory, a ratio which would be valid when the particle size no longer plays a part in the phenomenon:

$$S^* = \frac{S}{B} = 2.34 L^{+3/2} \tag{3}$$

where $L^+ = V/(qg)^{1/3} = V/V_c$; q = specific flow; g = gravity acceleration; V_c = critical velocity. Thus, by means of a suitable reduction to common parameters, they carry out a joint comparison of the empirical scour formulae, classifying them in three general types: Type I (non dimensionally homogenous) and Type II (dimensionally homogenous), the particle size is concerned; Type III (dimensionally homogenous), the particle size is not concerned and the limit ratios are constituted according to Veronese (1937).

Among the Type III limit formulations, Wu´s (1973) empirical non-dimensional ratios are $K = 2.11$; $\alpha = 0.235$; $\beta = 0.51$; $\gamma = 0$ and $\delta = 0$, obtained from prototypes and those of Lopardo et al. (1987) are $K = 2.50$; $\alpha = 0.25$; $\beta = 0.50$; $\gamma = 0$ and $\delta = 0$, obtained from models and prototypes. Table 1 shows the coefficients corresponding to theoretical formulations of plane and circular jets, as well as some limit scour formulations, expressed as a function of the variables S^* and L^+:

Table 1. Coefficients of general formulation $S^* = KL^{+n}$.

Author	K	n	Observation
Theoretical plane jet	2.70	2.00	Unlimited depth
Theoretical circular jet	6.40	1.00	Unlimited depth
Ramírez et al.	2.74	1.50	Plane. Maximum scour depth
Wu	1.79	1.47	Plane. Maximum scour depth
Lopardo et al.	2.10	1.50	Plane. Maximum scour depth

2.2 Mean dynamic pressure at the bottom of the plunge pool

The characterization of pressures by effect of a jet impingement was studied using different instrumentation (piezometers, pressure transducers, etc.), different means (air/air, air/water, water/water) and different typologies (rectangular nappe jet or circular jet). As far as the rectangular nappe typology is concerned, the results obtained by different authors may be summarized in the following general formulations, which represent the maximum dynamic pressure and their distribution:

$$\Delta p_{\max} = C\rho\left(\frac{V^2}{2}\right)\left(\frac{B}{h}\right) = \frac{Cq\gamma H^2}{h\left(2gH^3\right)^{1/2}} \tag{4}$$

$$\Delta p = \Delta p_{\max} e^{-k|x/h|^2} \tag{5}$$

where ρ and γ are the density and specific weight of the water. In Table 2 the coefficients C and k are shown, and a considerable dispersion of data can be seen, which is not surprising in view of the different natures of the tests.

The jet diameter D or the thickness jet B depends mainly on the type of discharge device (orifice jet impingement or overflow nappe impingement) and on the effect of the falling jet characteristics. Ervine et al.´s (1997) formulae include an estimation of the jet spread by the initial turbulent velocity in the case of the orifice jet impingement. This formula is valid for $h/D > 4\text{-}5$ and for H/H_b (jet plunge length / jet break-up length) < 0.5. The exponent k of the pressure distribution formula in the radial direction is 25 for non-effective water cushions (h/D < 4) and 30 for effective cushions.

However, there is still no estimation of jet spread in the case of overflow nappe, while it can be seen in any case that the models which are based on Froude´s similarity overestimate the pressures. For this reason, the determination of the thickness B is only carried out by gravitational considerations.

Table 2. Coefficients C and k of the general formulae of mean dynamic pressure

Author	C	k	Trial characteristics	Means	Observation
Cola	7.18	40.51	B = 12 – 24 mm h = 0.165–0.835 m V_0 = 1,3 – 4.8 m/s	Water Submerged jet with -out airation. Symetrical bidimensional	
Hartung & Hausler	5	19.6	Theoretical Jet. disintegration depth $y_k = 5B$	Water. Unlimited jet depth	If jet is considered rough, then coef. Erv. C=3.56; k=9.92
Beltaos:	8	42	B = 0.224 cm h/B = 45.5 - 68.2	Air. Bidimensional Different angles of impingement	Adjustment verification eq. Schauer &Eutis
Cui Guang et. al:	5.2–6.35 fects trial	12.56	Model without scale Prototype Q=80 m^3/s H =165 - 187m h = 32.5 - 54.9 m	Water. Bidimensional non symmetrycal (model of arch dam)	Possible ef- of scale of unknown
Armengou:	3.19	25	H = 1.8 - 5.5 m Q < 50 l/s h < 1.2 m V = 6 - 10.4 m/s *H/H_b = 0.4 - 2.73	Water. Non symmetrical Bidimensional Aerated jet	First values in starting of experimental facility trials
Puertas:	3.88	2	H =1.85 - 5.45 m Q < 86 l/s h = 0.08 – 0.80 m V = 6 - 10.4 m/s *H/H_b = 0.4 - 2.73	Water No symmetrical Bidimensional Aerated jet	The exponent of eq. (5) is m = 0.5
Ervine et. al:	38.4(1-Ci)(D/h)	25-30	H = 0.51 - 2.63 m Q < 63 l/s h = 0.10 - 0.5 m V = 4 - 25 m/s **H/H_b < 0.5	Water Circular jet Aerated jet	In D considers term of lateral spread by turbulence

*H_b = jet break-up length. Nappe flow: Horeni (1956): $H_b \approx 6\, q^{0.32}$

** Circular jet: Ervine et. al (1997): $H_b / D_0 F_0^2 = 1.05 / C^{0.82}$; $C = 1.14 T_u F_0^2$ = βi/(1+βi) = impingement initial air concentration; Tu = v´/V = jet turbulent intensity ; βi = $Qa/Q_w = K_1\left[1 - V_{min}/V\right]\sqrt{H/D}$; K_1 = 0.2 (smooth turbulent jet); K_1 = 0.4 (very rough turbulent jet); V_{min} ≈1 m/s = minimum velocity for air entrainment; Qa = air entrainment rate; Q_w = water discharge rate; D_0 = initial jet diameter; F_0 = initial Froude number;

$D = Dc + 2\varepsilon$; $\varepsilon = (1.14 T_u U_0^2 / g)\left[\sqrt{2H_b/(D_0 F_0^2)+1}-1\right]$; D = impingement jet; Dc = jet core; ε = lateral spread.

Puertas´s (1994) formulation covers the range of the jet break-up length $0.4 < H/H_b < 2.7$, but in a global way and for this reason, possibly underestimate the mean dynamic pressure coefficient $C_p = (\Delta p_{max})/V^2/2g$. Castillo (1998) carried out a new analysis with the data of Puertas and proposed different formulations of $C_p = f\ (h/B,\ H/H_b)$. On the other hand, the Puertas´s formulation is only valid for an effective water cushion:

$$h_e \geq 0.368 q^{0.50} H^{0,25} \tag{6}$$

If there is no effective cushion, the floor pressure is only reduced by the jet friction with the air and the friction effect with the pool depth. The pressure distribution equation (5) is very different from all the other formulations(m = 0.5 and k = 2) and this could reflect the possibility that the flow were mainly unidirectional (downstream) after impingement.

2.3 Analysis of the pressures for cushions below effective cushion

In the tests it has been observed that the sensor receives the impact in a non-uniform manner, because of the oscillation and break-up of the nappe. In this way, we expect that the results without an effective cushion to be more varied. An important fact, already noticed by Lencastre (1961) and verified by Castillo (1989) and Castillo et al. (1991) is that the maximum pressure fluctuation corresponded to small cushions of water and not to direct impact (h = 0); possibly one of the reasons might be the effect which is shown here; however, the reason already expressed by those authors should not be discarded: "by the small or null effect of small water cushions in the energy dissipation and by the advantage that a certain water cushion thickness offers for the development of turbulence". In order to know which proportion of distortion corresponds to a "measurement error" and which to the phenomenon of turbulence, new and more widely reaching tests should be carried out, so that a general law of pressures for below effective cushions could be drawn up.

Given that these phenomena are not considered in any of the formulations noted here, in the analysis as an approximate method for the case of direct impact, the formulation proposed by Moore (1943) is used, where the mean pressure is considered as the difference between the height of the discharge and the energy loss due to the friction effect of the pool depth y_p, so that:

$$\left(\frac{y_p}{y_c}\right)^2 = \left(\frac{y_1}{y_c}\right)^2 + 2\left(\frac{y_c}{y_1}\right) - 3 \tag{7}$$

where y_1 = contracted depth at the toe of the spill; y_c = critical depth.

The energy loss ΔE, deduced from momentum equation is:

$$\Delta E = \Delta z + (3/2) y_c - y_1 - \frac{V_m^2}{2g} \tag{8}$$

where $V_m = (V/2)(1+\cos\theta)$ = velocity corresponding to depth y_1; Δz = height from overflow crest to floor of plunge pool; $\cos\theta = 1.06/(\Delta z/ y_c + 3/2)^{0.5}$; θ = angle of the jet impingement.

3 RELATION BETWEEN LIMIT SCOUR AND MEAN DYNAMIC PRESSURE

If in the equation of Puertas (1994) the suitable transformations and calculations are carried out, the following formulation is obtained as a function of the variable $L^+ = V/(qg)^{1/3}$:

$$\frac{\Delta p_{max}}{B} = 2.74 L^{+3/2} F_b \tag{9}$$

$$F_b = \left(\frac{B^{0.5} H^{0.5}}{h} \right) = \frac{1}{(2g)^{0.25}} \frac{q^{0.5} H^{0.25}}{h} = 0.475 \frac{q^{0.5} H^{0.25}}{h} \tag{9.1}$$

It may be concluded that this formulation is similar to the scour formulations in the limit state of Ramirez et al. (1990); while the non-dimensional relationship F_b is close to unity. In this case $h = 0.475 q^{0.5} H^{0.25}$ is slightly greater than the effective cushion and establishes the requirements of plunge pool depth, according to the falling energy and the incident flow. It must be noted that h depends more intensely on the specific flow.
If Ervine et al.´s (1997) equation is expressed in the general form of a theoretical circular jet, we find that:

$$\frac{\Delta p_{max}}{D} = 29.27(1 - C) L^{+} F_e \tag{10}$$

$$F_e = \left(\frac{D^{1.33} H^{0.67}}{h^2} \right) = \left(\frac{2}{\pi g} \right)^{0.67} \frac{q^{1.33}}{h^2} = 0.162 \frac{q^{1.33}}{h^2} \tag{10.1}$$

where an aeration coefficient for jet C and an non-dimensional relationship Fe are included, the same which quantifies the action of flow impingement and the fall energy, with the water cushion requirements. It can be seen that the flow impingement actions as well as the height of fall are much more intense than in the case of the plane jet; as well as the needs of the water cushions to damp the action.

In Figure 2 the theoretical relationship of plane and circular jets are shown; ratios (S/B) vs. L^+ for the limit scour formulations of Ramírez et al. (1990), Wu (1974) and Lopardo et. al (1987); and the ratios ($\Delta p_{max}/B$) vs. L^+ of the mean dynamic pressures formulations of Puertas (1994) and Ervine et. al (1997), including different values of Fb, C y Fe.

The pressures for the theoretical circular jet constitute a higher envelope up to a value of the jet entry velocity in the order of around 2.40 times the critical velocity ($L^+ = 2.40$), and from there onwards, the envelope of maximum pressure corresponds to theoretical plane jet.

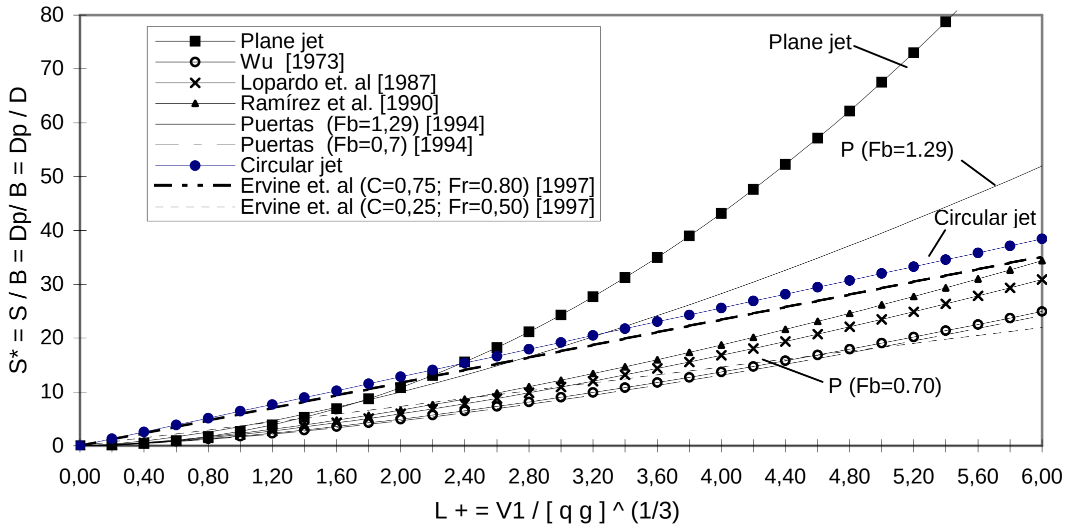


Figure 2. Relationship among mean dynamic pressure and limit scour formulation.

The extreme limits of non-dimensional relationship Fb = 1.29 and 0,70, constitute higher and lower envelope in pressures for the formulations of limit scour; showing, as it was expected that the maximum dynamic pressures are always reached when there is a minimum water cushion and vice versa; that is to say, the minimum dynamic pressures will be obtained when the height of the water cushion is greater, coinciding in this case with Wu´s (1973) limit scour formulation; while with Lopardo et al.´s (1987) limit scour formulation, the approximation is carried out for an intermediate value of the above-mentioned non-dimensional relationship.

In Figures 3, 4 and 5, variations of the water cushion, total energy and energy dissipation are presented. These are calculated: from Moore´s (1943) classic formula; the mean dynamic pressure for different values of F_b according to Puertas (1994) and F_e according to Ervine et. al (1997) and, finally Ramírez et al.´s (1990) limit scour depth. The analysis was carried out for q = 20 m^2/s and H = 25 - 250 m.

As to the depths of the water cushion (Fig. 3), it can be seen that the smallest depths of water cushion correspond to the contracted depth y_1 and which, in theory ought always to contain the greatest amount of energy (or produce the least energy dissipation). As is logical, the depth y_1 decreases as the height of the fall increases. However, it should be noticed that the real water cushion for the case of direct impact corresponds to the pool depth y_p; the same value is slightly greater than the effective cushion calculated from Puertas´s formula, while it becomes the same from H = 170 m. It should be noticed that the pool depth does not completely surround the jet and so there is not a dissipation of energy by the effects of the jet diffusion.

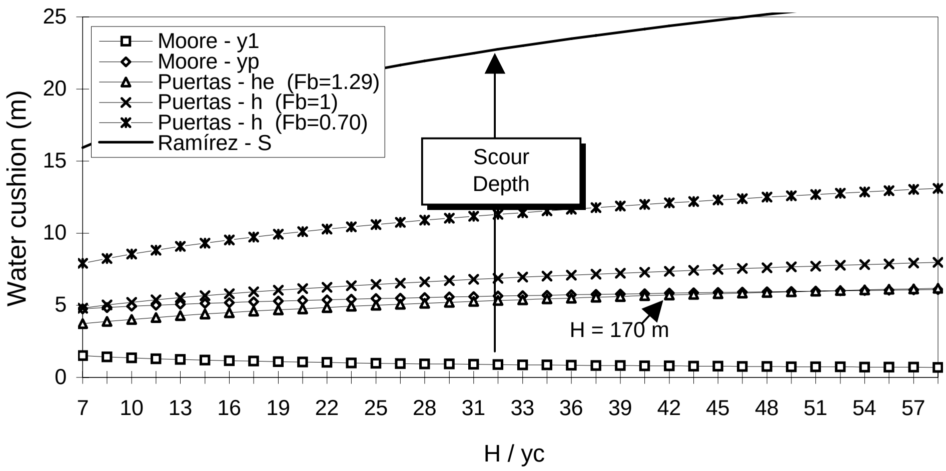


Figure 3. Water cushion depth: q = 20 m^2/s; yc = 3.442 m; H = 25 – 250 m.

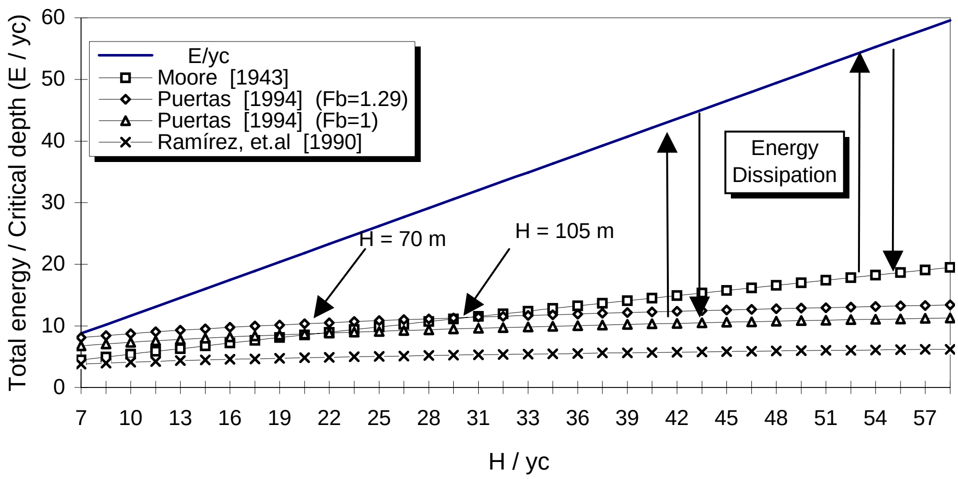


Figure 4. Energy dissipation: q = 20 m^2/s; yc = 3.442 m; H = 25 – 250 m.

In Figures 4 and 5 it can be seen that with Moore´s formulation higher levels of energy dissipation are reached than those obtained with Puertas, up to H = 70 m for F_b = 1.29 and H =105 m for F_b = 1. This would show that for the flow analysed, part of the jets would reach the floor in a more or less compact manner up to the height of 70 m, and then from this point an increase of air entrainment and break-up jet would be accentuated, thus bringing about an energy dissipation in the air which is not considered in Moore´s formulation and which, in Puertas´s formulation, since this phenomenon is implicitly registered in the tests, the above-mentioned loss is taken into account in some way.

As it is logical to expect, the greater water cushions values are obtained with Ramírez et al.´s scour limit formulation and thus will always contain the least amount of energy (Fig. 4) or the production of greatest energy dissipation (Fig. 5)

The water cushion calculations obtained by means of Puertas´s formulation increase proportionally with the reduction of the non-dimensional relationship F_b, containing less energy (Fig. 4) and producing a greater amount of energy dissipation (Fig. 5).

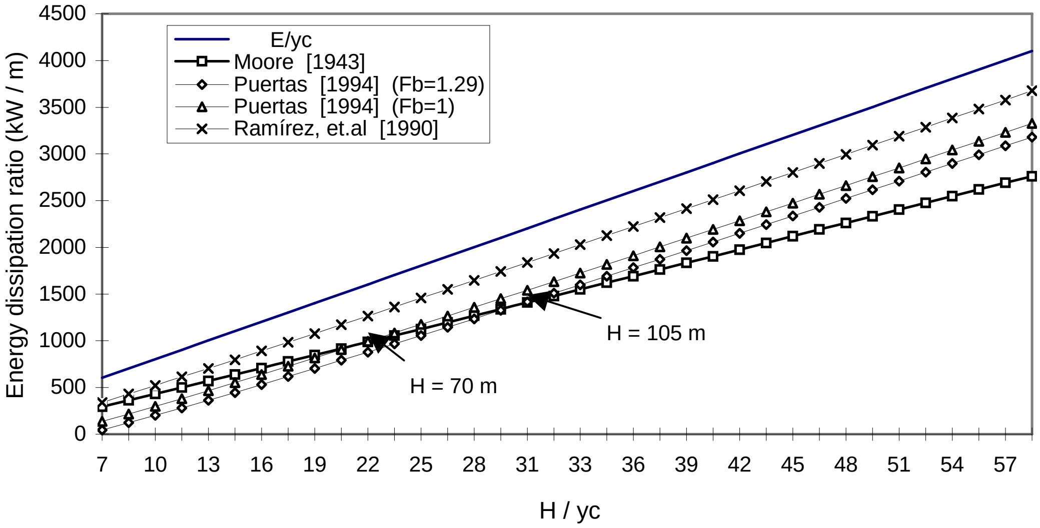


Figure 5. Energy dissipation relation: q = 20 m^2/s; yc = 3.442 m; H = 25 – 250 m.

4 EROSIVE POWER OF THE JET IMPINGIMENT

Since the choice of a typology for the dissipation of energy in a scour hole or with a rigid floor will depend on the geological / geotechnical / economic and environmental conditions, it is interesting to find a direct relationship between the two design typologies, in practical terms of use. Thus, if it is known that a design in scour hole requires a scour height S, what equivalence would there be with a rigid floor (or rock floor) typology with a water cushion h?. Which practical mechanisms should be borne in mind in terms of pressure fluctuations and turbulence intensities?. The answer to these questions would be carried out to the choice of one or another type of design.

Following Annandale´s work (1995), the “energy dissipation ratio” constitutes a parameter that reasonably represents the relative strength of the fluctuating disturbance and can be easily calculated. Thus, if the energy loss is ΔE and the specific flow q, the “energy dissipation ratio” per unit of width of flow is expressed as:

$$P = \gamma q \Delta E \tag{11}$$

The relationship between “energy dissipation ratio” P and the material resistance can be expressed as the function:

$$P = f(K_h) \tag{12}$$

at the resistance threshold. If $P > f(K_h)$, the resistance threshold is exceeded, and the material would be expected to fail. In our case, the required material constitutes a rigid plate or the rock which resists the pressure fluctuations which water cushion *h* has been unable to dissipate. Thus, an alternative design for a basin with rigid floor (plate or rock) and water cushion height *hp* could be analysed with another basin and cushion height *hm* but this in turn is related to the basic typology which constitutes the design of a scour hole with height *S*; the same height for the design conditions constitutes the basin with minimal resistance, since it has allowed the greatest energy dissipation ratio to develop. However, the difference in the energy production ratio DP between the scour hole P_s and the basin with a rigid floor P_h, constitutes the net energy dissipation ratio "incremental energy dissipation" or equivalently of pressure fluctuations which the plate or rock floor of the basin would have to resist; thus:

$$DP = P_s - P_h \tag{13}$$

In Figure 6 the "incremental energy dissipation" (or the differences of energy production relation), can be found, taking as a reference of calculation the Ramírez et al.´s formulation. Moore and Puertas (F_b = 1.29, F_b = 1.00 y F_b = 0.70) formulations are analysed and it can be seen that the rigid plate design with Moore criteria, would be the least stressed for H < 45 m; however, would be the most stressed for H > 105 m. In the other cases, behaviour is as expected; that is to say, the most stressed correspond to the design criteria with a strict effective water cushions (F_b =1.29).

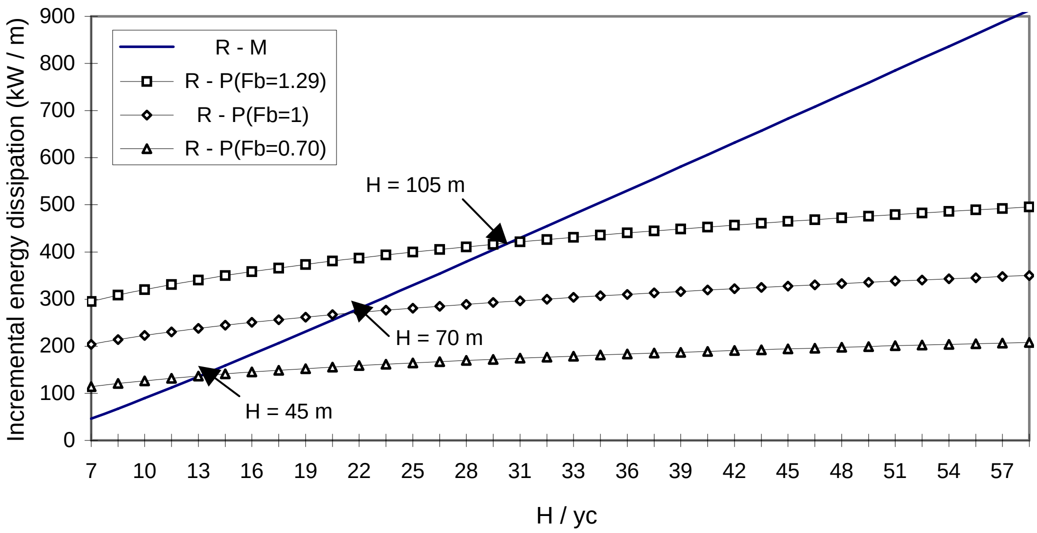


Figure 6. Incremental energy dissipation *DP*: q = 20 m^2/s; yc = 3.442 m; H = 25 – 250 m.

5 CONCLUSIONS

A combined analysis is presented of the limit scour and mean dynamic pressure formulation, showing that they correspond to the same type of formulation. Different analysis of parametric sensitivity were carried out, according to the height of fall.

A practical parameter is presented in order to estimate the necessary resistance which the rigid basin floor would have to resist by the power of water jet "incremental energy dissipation", which allows the analysis of the different actions in the plunge pool. In this sense, a study should be carried out of the correlation between energy dissipation and the rigid floor material resistance, perhaps following the guidelines set forth by Annandale, for the erosionability phenomena.

It would be important to complement the studies with the wall flow on the bottom and the pressures for water cushions smaller than effective cushions.

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