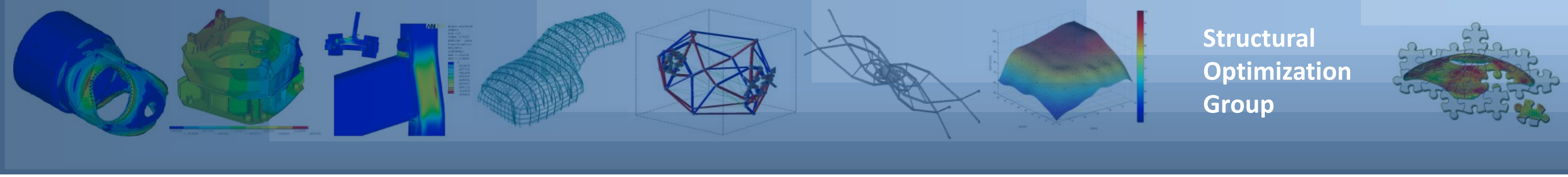
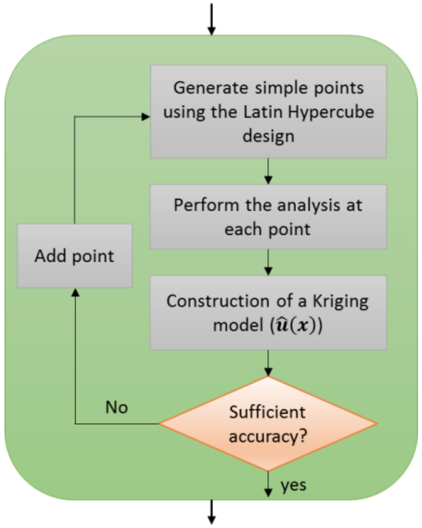
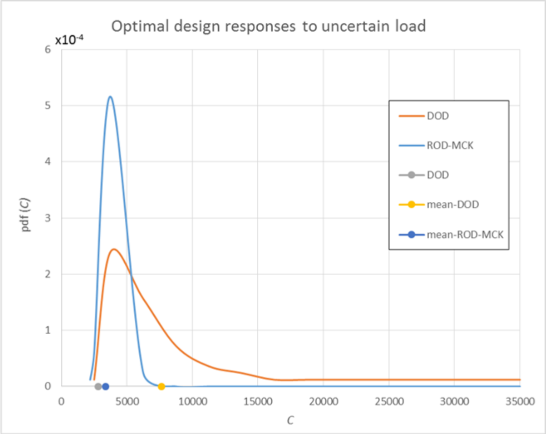


Robust Topology Optimization of Structures using Kriging Models





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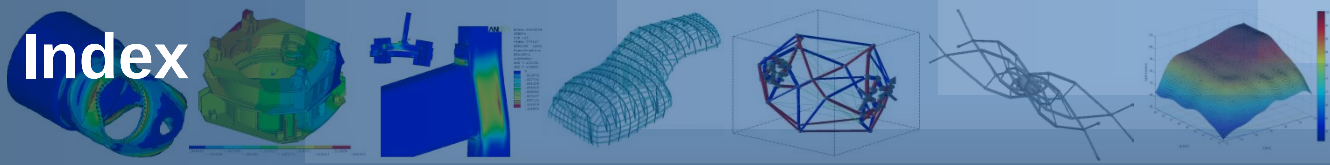

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Introduction

- Structures used in real world should consider the effect of an uncertainty environment.
- Design under uncertainty:
  - Reliability-Based Design Optimization (RBDO). Minimum failure probability.
  - Robust Design Optimization (RDO). Solution insensitive to uncertainties.
- Robust Topology Optimization (RTO), is a combination between Robust Design Optimization (RDO) and Topology Optimization (TO).
- Some works about RTO under uncertainty in loading:
  - Chen et al. (2010).
  - Dunning and Kim (2011; 2013).
  - Zhao and Wang (2014-a; 2014-b).

Robust topology optimization: Formulation

min.	$C(\mathbf{u})$	**min.**	$E[C(\mathbf{u},\boldsymbol{z})]$
Subject to:	$\mathbf{K}(\boldsymbol{\rho})\mathbf{u}(\boldsymbol{\rho}) = \mathbf{f}$	Subject to:	$\mathbf{K}(\boldsymbol{\rho},\boldsymbol{z})\,\mathbf{u}(\boldsymbol{\rho},\boldsymbol{z}) = \mathbf{f}(\boldsymbol{z})$
	$V \leq V_{\max}$		$V(\mathbf{z}) \leq V_{\max}$
	$0 \leq \boldsymbol{\rho} \leq 1$		$0 \leq \boldsymbol{\rho} \leq 1$

$C(\cdot)$: compliance,

$\mathbf{u}$: displacement field,

$\mathbf{K}$: stiffness matrix,

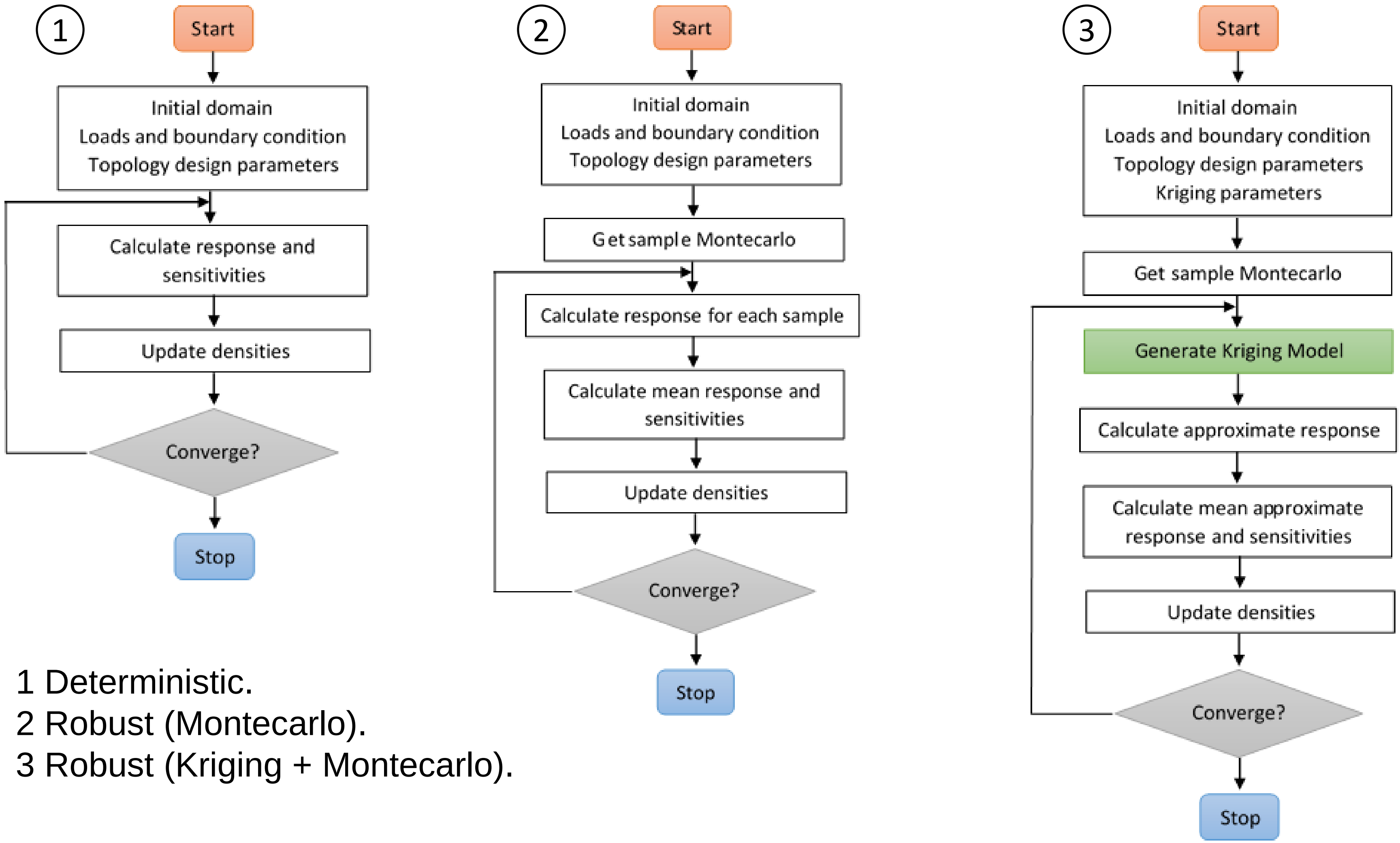
$\mathbf{f}$: load vector,

$\boldsymbol{\rho}$: densities vector,

$\boldsymbol{z}$: uncertainty variables,

$E[\cdot]$: expected value.

Robust topology optimization: algorithm



1 Deterministic.
2 Robust (Montecarlo).
3 Robust (Kriging + Montecarlo).

SIMP method

- Density based method (Bendsøe 1989; Rozvany et al. 1992)

$$0 \leq \rho_e \leq 1.$$

- Penalization for intermediate densities:

$$E_e(\rho_e) = E_{min} + (E_0 - E_{min})\rho_e{}^p.$$

$E(\boldsymbol{x})$ Young modulus updated,
E_0 Young modulus solid material,
E_{min} Young modulus for void material,
p: penalization factor.

- Density filter is used to avoid mesh dependent solution and checkerboard patterns.
- The topology optimization problem is solved by means of a standard optimality criteria method (Sigmund 2001).

Uncertainty: Quantification

- In this work are considered independent random variables in loading.
  - Magnitude, Direction, Position.
- A random variable is defined by a probability density function (pdf).

pdf	Parameters
Beta	a: first shape parameter , b: second shape parameter
Chisquare	ν: degrees of freedom
Exponential	μ: mean
Gamma	a: shape parameter, b: scale parameter
Inverse Gaussian	μ: scale parameter, λ: shape parameter
Longnormal	μ: log mean, σ: log standard deviation
Normal	μ: mean, σ: standard deviation
Poisson	λ: mean
Uniform	a: lower endpoint, b: upper endpoint
Weibull	a: scale parameter, b: shape parameter

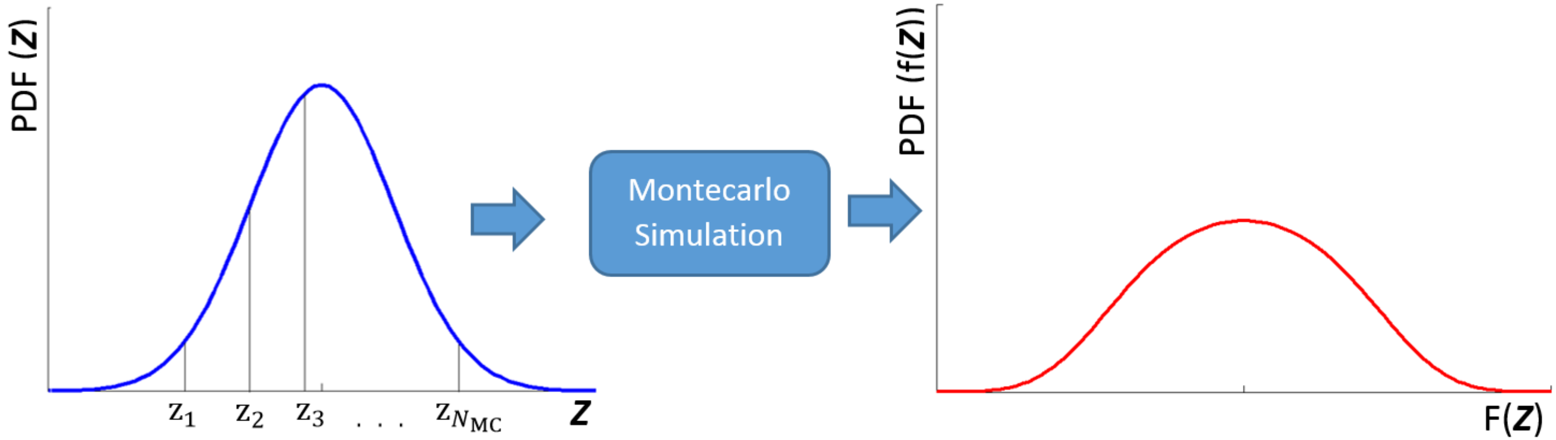
Uncertainty: Propagation

- Performance expected value μ_f is defined as

$$\mu_f = E[C(\boldsymbol{u},\boldsymbol{z})] = \int C(\boldsymbol{u},\boldsymbol{z})\,\mathrm{pdf}(\boldsymbol{z})\,dz$$

- This integral can be difficult to evaluate.
- Therefore approximate methods are used:
  - Simulation methods: Montecarlo (MC), Quasi-MC, Latin Hypercube,…
  - Expansion methods: collocation method, perturbation method,…
  - FORM, SORM.
  - Meta-models: response surface method, Kriging methods,…
  - Approximate integration: Univariate Dimension Reduction (UDR), Bivariate Dimension Reduction (BDR), …
- The Montecarlo (MC) method is used in this work.

Montecarlo Methods



$$\mu_f = E[C(\boldsymbol{\rho}, \boldsymbol{z})] \approx \frac{1}{N_{\mathrm{MC}}} \sum_{i=1}^{N_{\mathrm{MC}}} C(\boldsymbol{\rho}, z_i)$$

- The accuracy of estimate is good if N_{MC} is large (>10000).
- The computational cost is proportional to N_{MC}.

Kriging Models (1)

- Kriging Models are interpolation models.
- They are used to surrogate a true response $f(\boldsymbol{x})$.
- Reduce the computational cost

$$\hat{f}_l(\boldsymbol{x}) = F(\beta_{:,l}, \boldsymbol{x}) + Z_l(\boldsymbol{x}) \qquad l = 1, \dots, q$$

Regression model

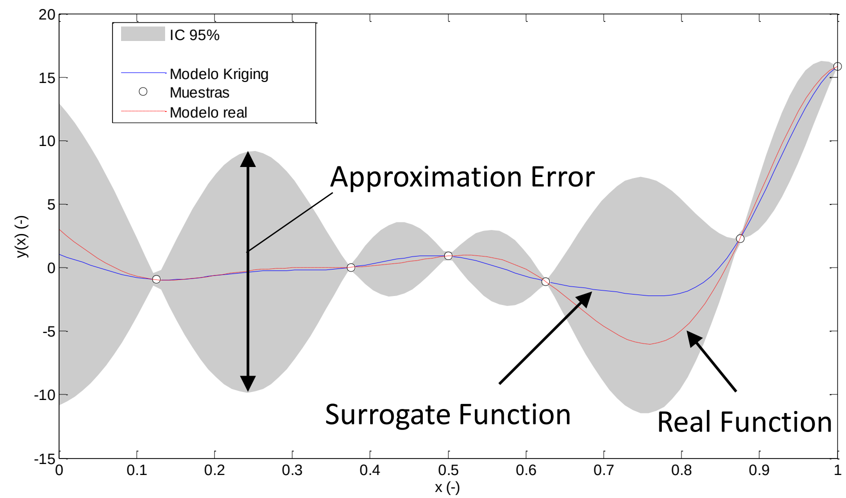
$$F(\beta_{:,l}, x) = \sum_{i=1}^{p} \beta_{i,l} f_i(x)$$

Stocastic process

$$E[z_l(w), z_l(x)] = \sigma_l^2 \Re(\theta, w, x),$$

$$E[z_l(x)] = 0,$$

$$l = 1, \dots, q$$



- Design steps:

1. Latin Hypercube Design (size N_K)

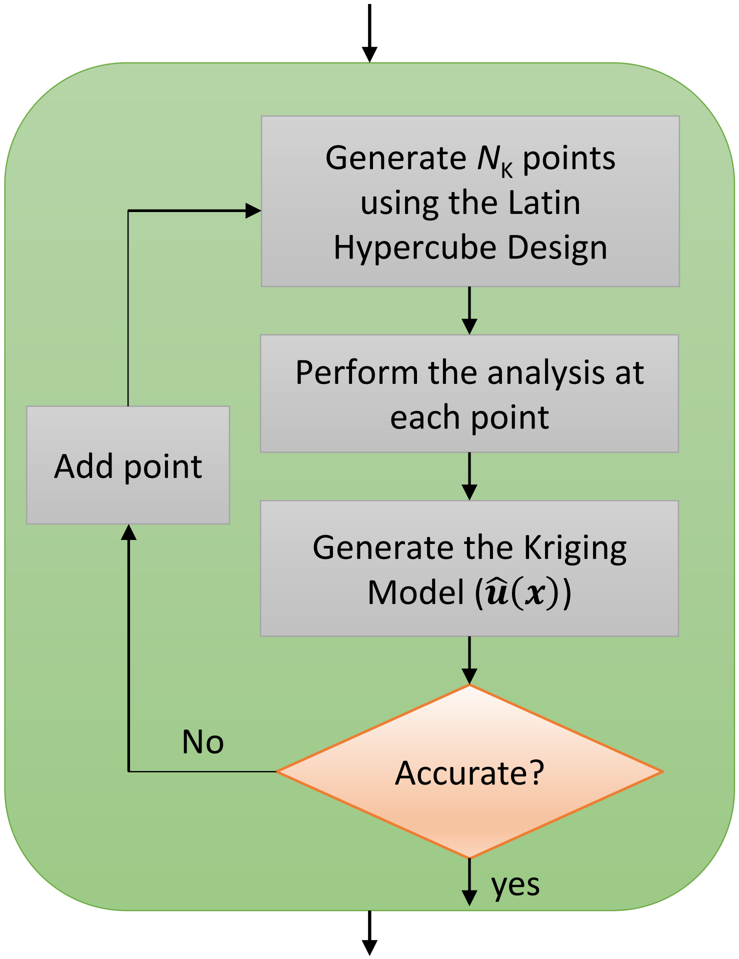
$$\mathbf{S} = [\mathbf{s}_1\ \mathbf{s}_2\ \dots\ \mathbf{s}_{N_K}].$$

2. Evaluate structural response (displacement)

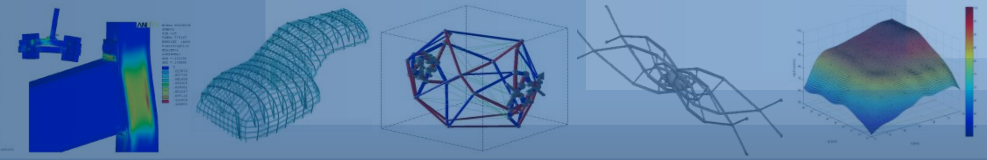
$$\mathbf{u} = [\mathbf{u}_1\ \mathbf{u}_2\ \dots\ \mathbf{u}_{N_K}].$$

3. Generate the Kriging Model

$$\hat{\boldsymbol{u}}(\boldsymbol{x}) \approx f(\mathbf{S}, \mathbf{u}).$$



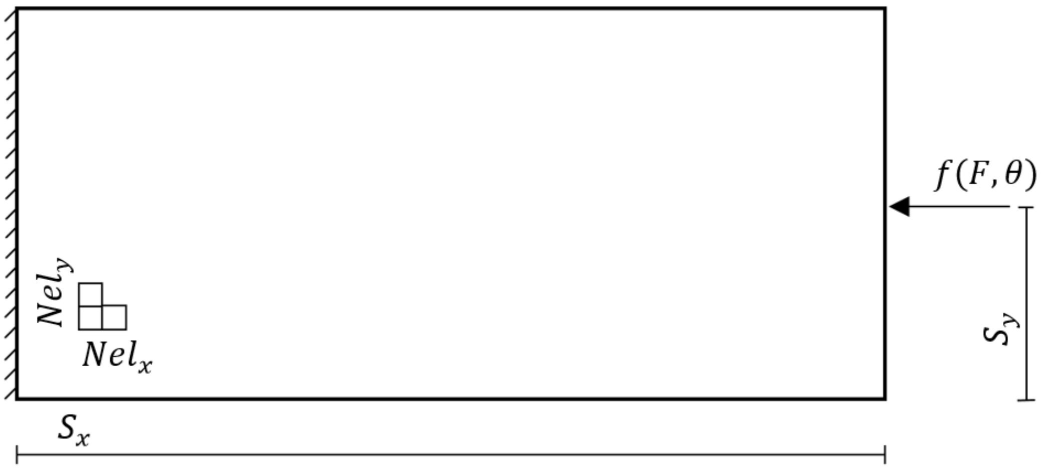
Examples:



Structural Optimization Group

- Elasticity module:
  - $E_o/E_{\min}$: 1/10^{-4},
  - Poisson's ratio: 0,3.
- Finite elements:
  - 4-node bilinear,
  - Unit sized elements.
- SIMP parameters:
  - Penalization factor: p = 3,
  - Filter radio: r = 2.
- Kriging Model
  - Regression function: first order polynomial.
  - Correlation function: exponential.

Example 1: Cantilever beam (1)

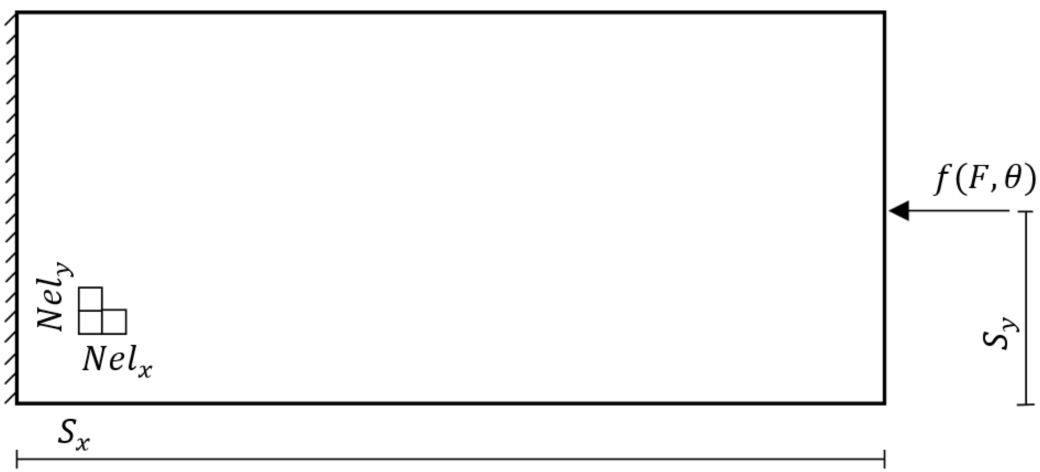


$$V_f/V_o = 0{,}3$$

f: Punctual load.
F: Magnitude.
θ: Direction.
S_x: Load position on x axis (0-1).
S_y: Load position on y axis (0-1).

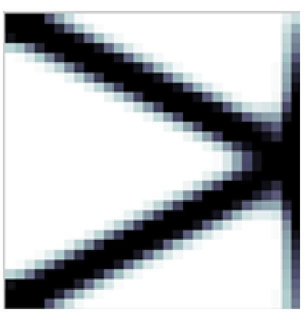
Case 1: Uncertain load position.
Case 2: Uncertain load magnitude.
Case 3: Uncertain load direction.

Example 1: Cantilever beam (2) Uncertain load position

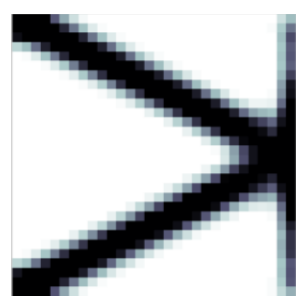


Case 1:

V_f/V_o = 0,3;
F : 1;
θ : +90º;
S_x : 1.0, S_y : Normal (0,5 0,17);
Nel_x : 30; Nel_y : 30;
N_{MC} : 10000; N_K : 10.

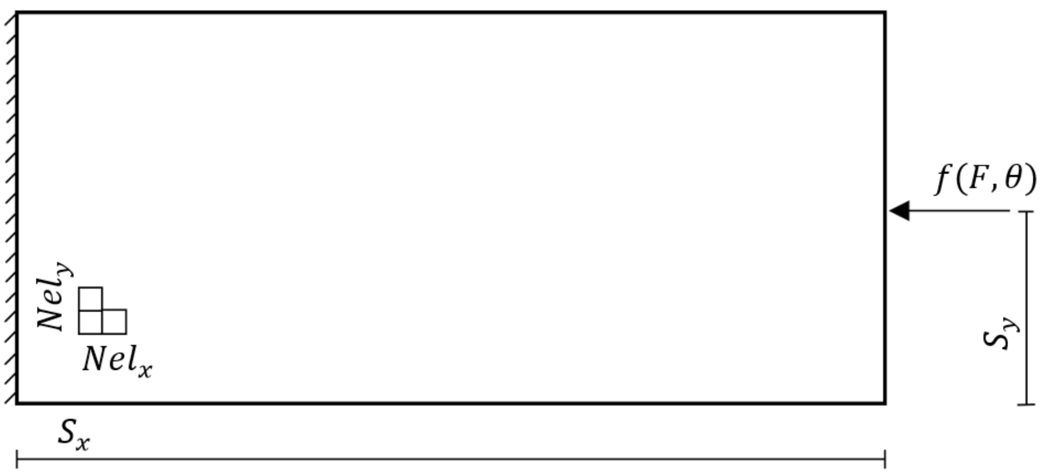


Montecarlo:
E[C] = 34,83



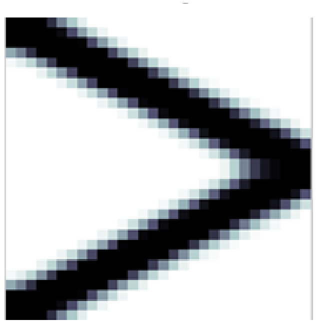
Montecarlo-Kriging:
E[C] = 35,27 (+1,2 %)

Example 1:
Cantilever beam (3) Uncertain load magnitude

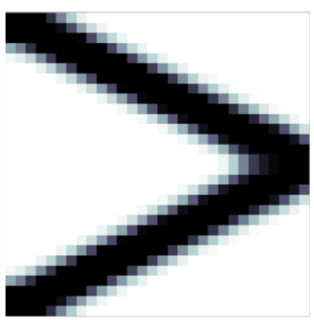


Case 2:

V_f/V_o = 0,3;
F : Normal (1 0,033);
θ : +90º;
S_x : 1,0, S_y : 0,5;
Nel_x : 30; Nel_y : 30;
N_{MC} : 10000; N_K : 6.

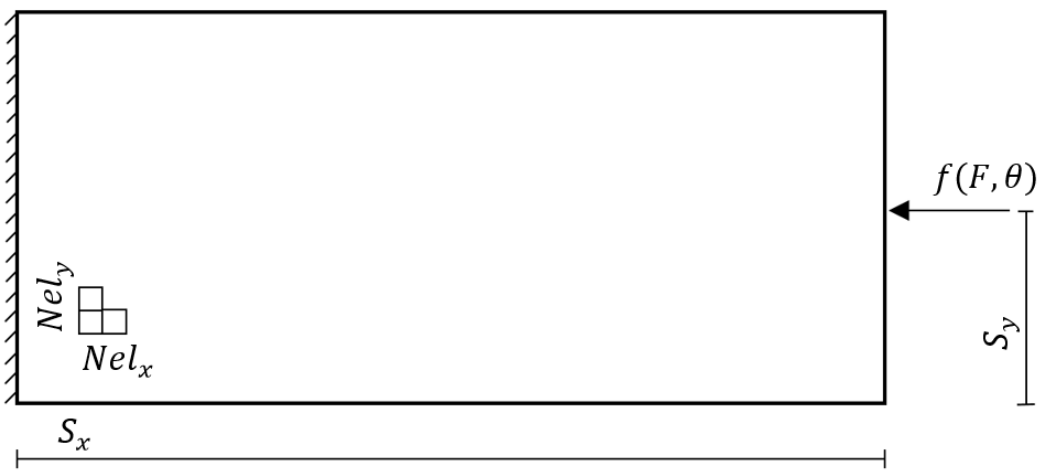


Montecarlo:
E[C] = 26,65



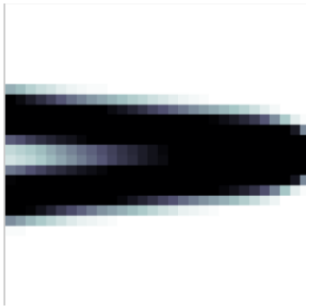
Montecarlo-Kriging:
E[C] = 26,60 (-0,2 %)

Example 1: Cantilever beam (4) Uncertain load direction

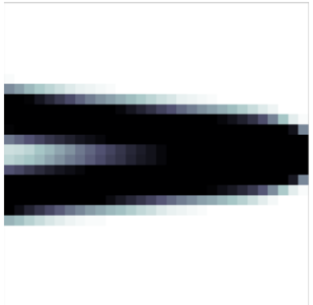


Case 3:

$V_f/V_o = 0,3$;
F : 1;
θ : Normal (0º 5º);
S_x : 1,0, S_y : 0,5;
Nel_x : 30; Nel_y : 30;
N_{MC} : 10000; N_K : 6.

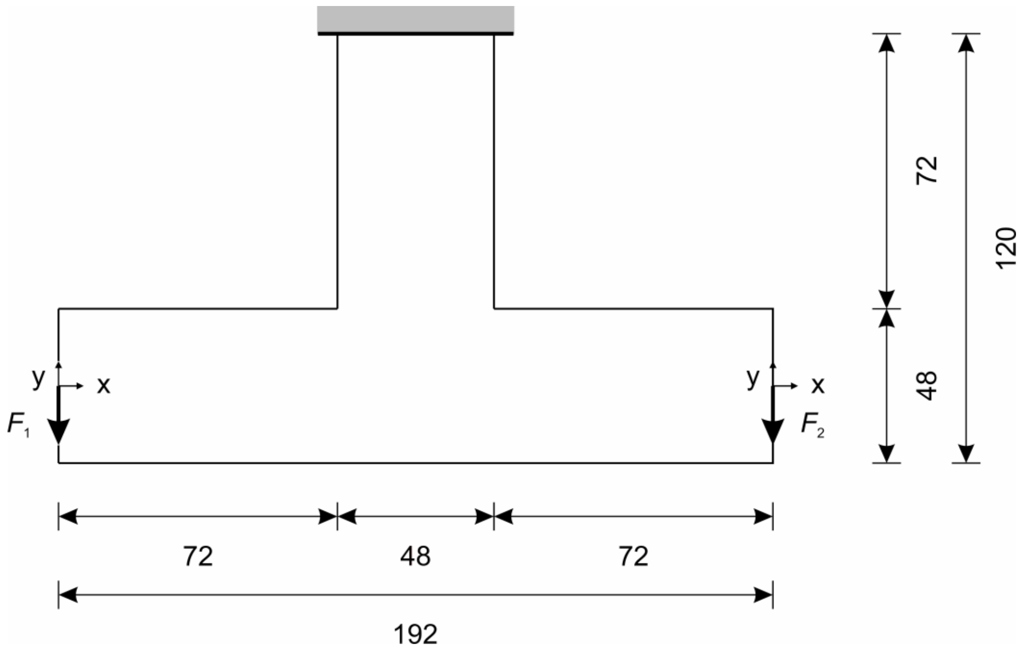


Montecarlo:
E[C] = 6,04



Montecarlo-Kriging:
E[C] = 6,05 (-0,2 %)

Example 2: Inverted T (1)



$N_{MC} = 10000,$

$N_K = 6,$

$V_f / V_o = 0,5.$

Uncertain loads F_1 F_2,

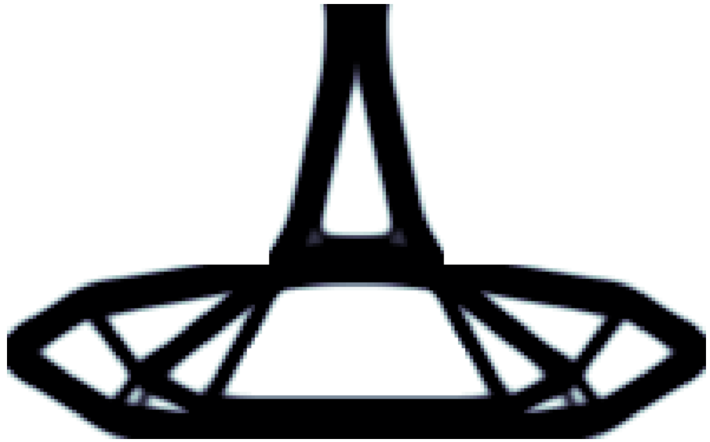
Normal ($\mu_F = 5,0$ $\sigma_F = 0,5$),

Normal ($\mu_\theta = -90^o$ $\sigma_\theta = 14,3^o$).

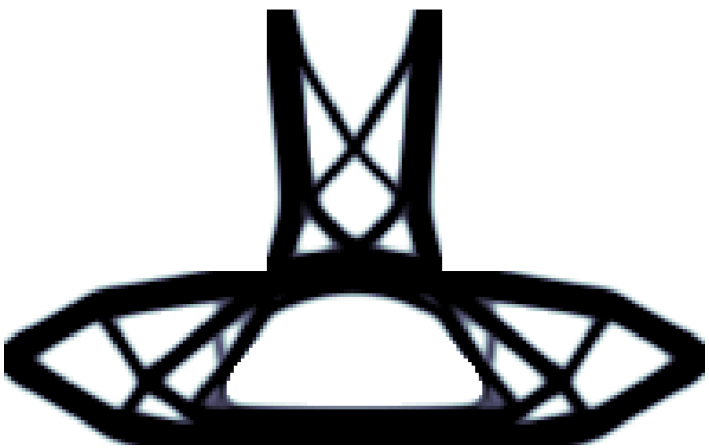
12672 bilinear elements,

2570 dof.

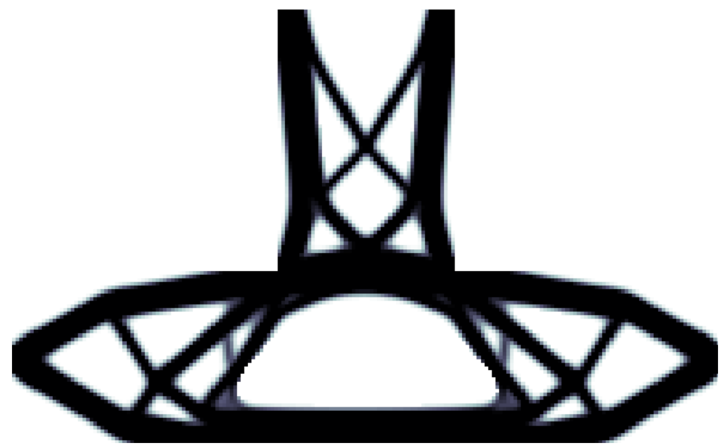
Example 2: Inverted T (2)



Deterministic design C = 2786,91



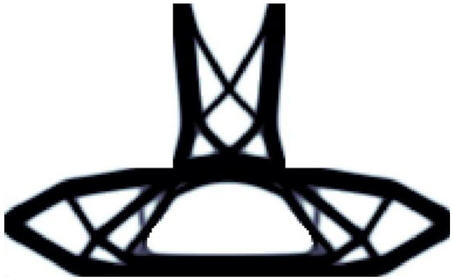
Robust design (MC), E[C] = 3341,70.
t_i = 99388 s



Robust design (MCK), E[C] = 3328,20 (-0,40 %).
t_i = 44 s (-99,95 %)

Example 2: Inverted T (3)

Deterministic design



Robust design

Deterministic design

C = 2786,91

Robust design with uncertain load

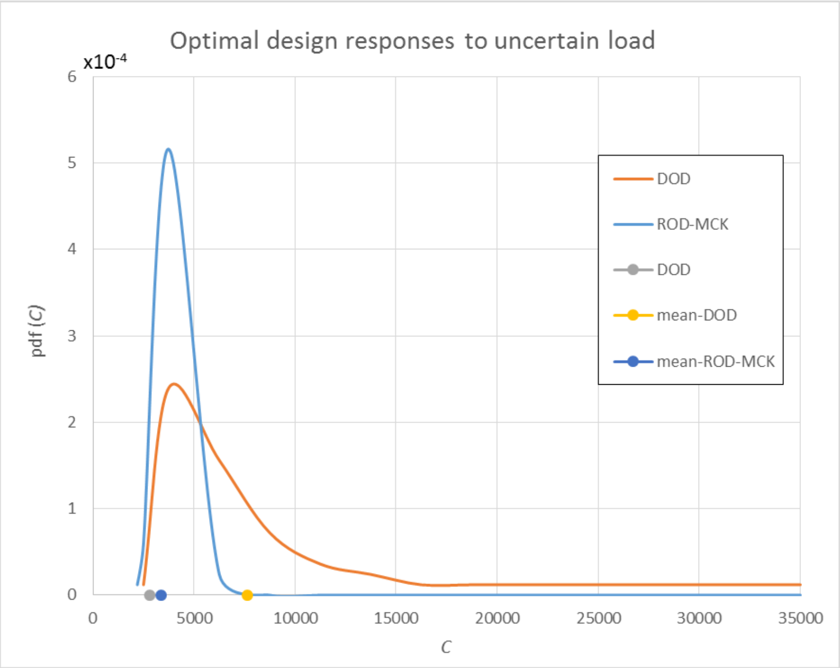
E[C] = 3376,82 SD[C]= 701,74

Deterministic design with uncertainty load

E[C] = 7657,80 SD[C] = 6519,0

E[·] : expected value

SD[·] : standard deviation



Conclusions

- A general methodology for topology optimization with uncertainty is presented in this work.
- Loading uncertainties are considered in magnitude, direction and position, like independent random variables.
- Montecarlo method is used to propagate the uncertainty to the response and a Kriging Model is used to reduce the computational cost.
- The proposed methodology (MCK) is accurate and very efficient. The computacional cost is much lower than standard Montecarlo method.

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Thanks for your attention

Gracias por su atención

Obrigado pela atenção