Metamodel-based multi-objective robust design optimization of structures

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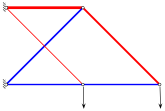
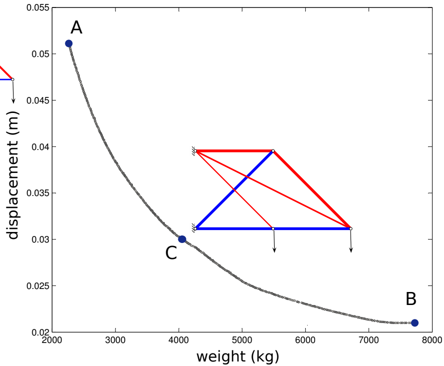
Outline

Multi-objective optimization
Pareto frontier

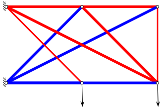
The goal of Multi-objective optimization is to find a collection of solutions that form trade-offs between the multiple objectives.





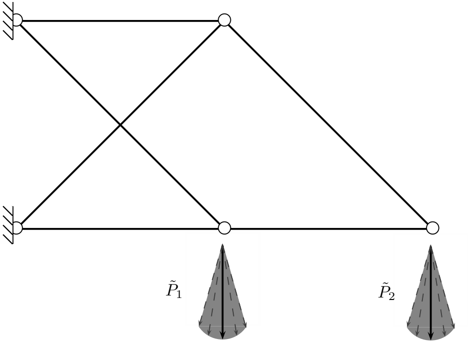
How to solve MOOP problem

- Scalarizing methods
  - weighted-based methods
  - constraint-based methods
- Multi-objective evolutionary algorithms (MOEAs)



Multi-objective optimization
Robust solutions

In real-world optimization problems, the optimal performance obtained using conventional deterministic methods can be dramatically **degraded** in the presence of **sources of uncertainty**.



Sources of uncertainty

- Applied loads
- Spatial positions of joints
- Section properties
- Material properties

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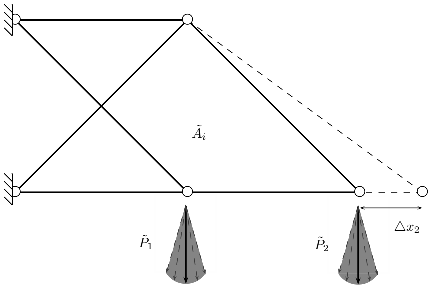

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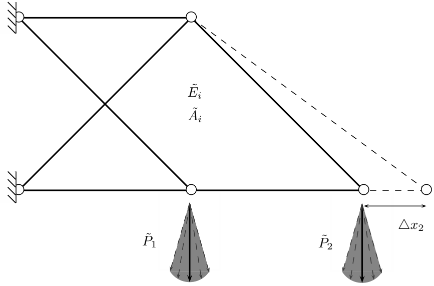

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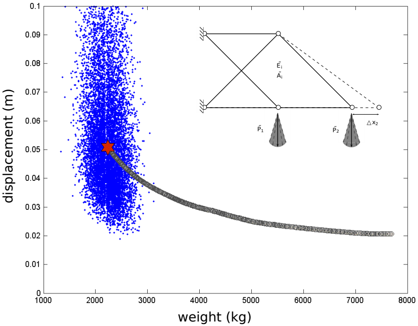

Sources of uncertainty

- 10% variability in Young's modulus
- 5% variability in Areas
- 1% variability in position of joint 2
- $\pm 5^\circ$ variability in the orientation of loads

Multi-objective Robust optimization
Expectation-variance based approach

Probabilistic approach

The structural robustness is evaluated by the measure of the performance variability around the expected value.

$$\begin{aligned} &\min_{\mathbf{x}\in\mathbb{R}^n} \{\mu_{f_1(\mathbf{x})} + k\sigma_{f_1(\mathbf{x})}, \ldots, \mu_{f_n(\mathbf{x})} + k\sigma_{f_n(\mathbf{x})}\}\ (n \geq 2, k \geq 0) \\ &s.t.\ \ \mu_{g_j(\mathbf{x})} + k\sigma_{g_j(\mathbf{x})} \leq 0 \quad j = 1, \cdots, m_i \\ &\qquad \mathbf{x}^{\text{lower}} \leq \mathbf{x} \leq \mathbf{x}^{\text{upper}} \end{aligned} \tag{1}$$

- Varying the value of k, different levels of robustness can be obtained.
- The nested relationship between uncertianty quantification and optimization can lead to an intractable problem.

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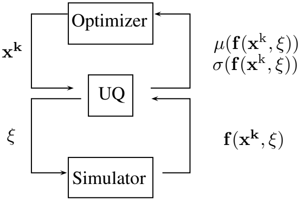
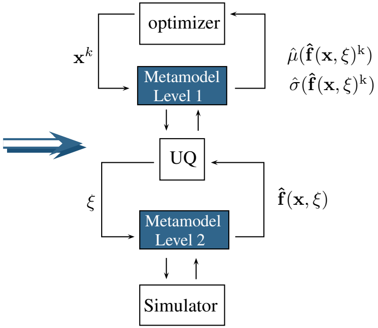
Multi-objective Robust optimization
Metamodel-based approach

Objective

To efficiently include the uncertainty quantification in the optimization process using mathematical approximations called metamodels.

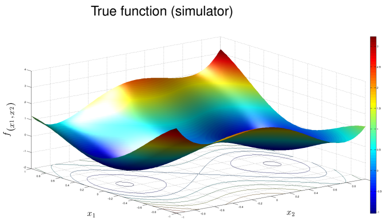


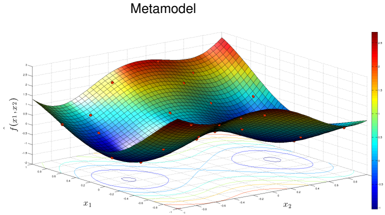


$\mathbf{x}^k$ $\Rightarrow$vector of optimization variables at iter. k.
ξ $\Rightarrow$ vector of random variables.

Meta-modelling

Consist of replacing a computationally expensive simulation model by a mathematical approximation which is much faster to evaluate.





1. To sample the function to be predicted.
2. To create a mathematical approximation using statistical considerations.
3. To evaluate the accuracy of the mathematical model.

Meta-modelling
Kriging models

Kriging model

Kriging models assume that the simulator can be approximated by a sample path of a Gaussian stochastic process $\mathscr{G}(\mathbf{x})$

Prior mean

$$E[\mathscr{G}(\mathbf{x})] = \boldsymbol{f}(\mathbf{x})^{\mathrm{T}} \boldsymbol{\beta}$$

Prior covariance function

$$Cov[\mathscr{G}(\mathbf{x}), \mathscr{G}(\mathbf{x}')] = \alpha^2 \exp(\sum_{\mathrm{i}=1}^{\mathrm{n}} -\frac{|\mathrm{x_i} - \mathrm{x_i'}|^s}{\phi_{\mathrm{i}}})$$

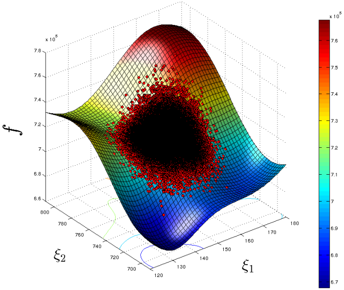
The parameters $\boldsymbol{\beta}$, α^2 and ϕ are unknown a priori and are determined from the set of simulator responses

$$\hat{y}(\mathbf{x}) \equiv \mathrm{E}[\mathscr{G}(\mathbf{x})|\mathscr{Y}] = \boldsymbol{f}(\mathbf{x})^{\mathrm{T}}\hat{\boldsymbol{\beta}} + \mathbf{r}(\mathbf{x})^{\mathrm{T}}\mathbf{R}^{-1}(\mathscr{Y}^{\mathrm{T}} - \mathbf{F}\hat{\boldsymbol{\beta}})$$

Kriging-based Multi-objective Robust Optimization (K-MORO)

The proposed method consists of two-stage framework:

1. To replace the expensive simulator by a Kriging model to carry out the Monte-Carlo analysis.
2. To approximate the statistical moments of both the objective functions and the constraints on the design domain.



⟺ UQ (Monte-Carlo)

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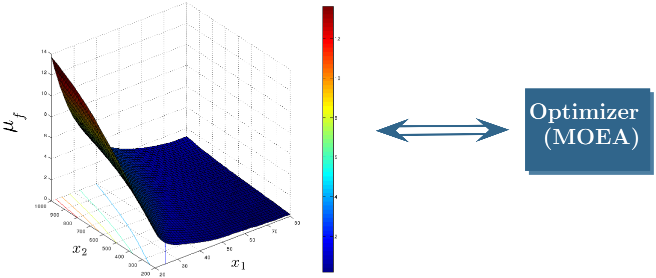


Problem definition
two-bar truss structure

(Messac et al., 2002)

$$\min_{d,H} \{\mu_{volume} + k\,\sigma_{volume},\ \mu_{deflection} + k\,\sigma_{deflection}\}$$

$$s.t.\ g_1 = \mu_S + k\,\sigma_S \leqslant S_{max}$$

$$g_2 = \mu_S + k\,\sigma_S \leqslant S_{crit}$$

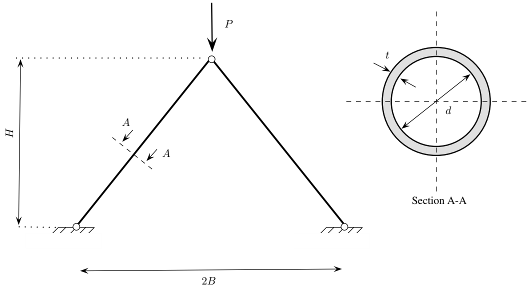
$$20 \leq d \leq 80,\quad 200 \leq H \leq 1,000$$

Design variables:

- Structure height (H).
- Cross section diameter (d).

Random parameters:

- Vertical force (P).
- Structure width (B).
- Elastic modulus (E).
- Member thickness (t).



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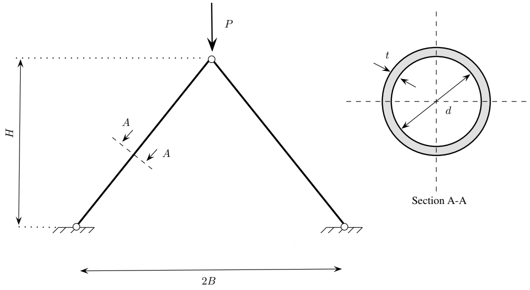
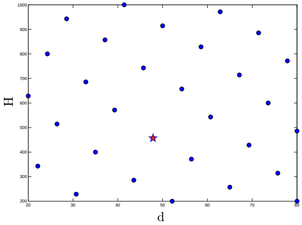
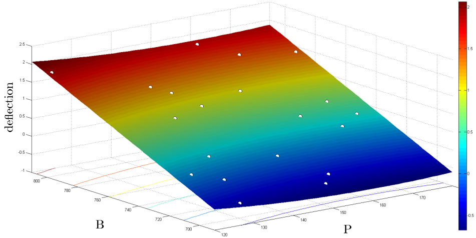


Latin Hypercube Sampling
design variable domain (H, d)
(30 points)



Latin Hypercube Sampling
random variable domain (P, B, E, t)
(20 points)



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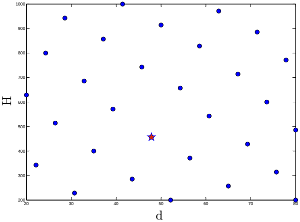

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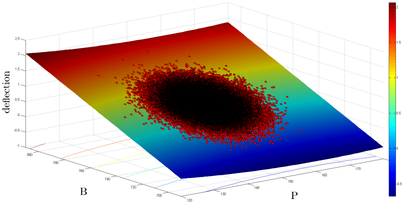
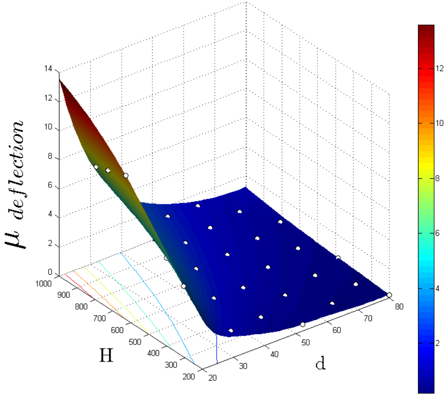


For each design point (H_i, d_i) statistical moments are obtained using Monte Carlo simulation (10^5 points)

$\mu_{deflection}$

$R^2 = 0.998$



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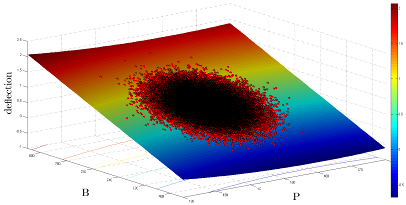

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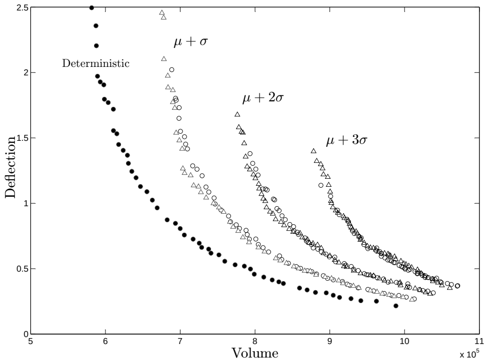
Multi-objective optimization
Robust Pareto fronts & contours of volume

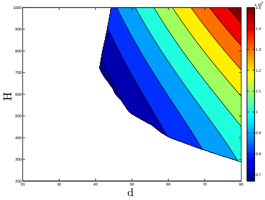
Robust Pareto fronts (determistic, σ, 2σ, 3σ)



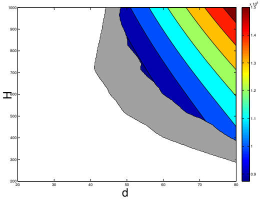
nested approach(○), metamodel-based approach (△).

Feasible region

σ



3 σ



Concluding remarks

The use of Metamodels **breaks the nested relationship** between the optimization and the uncertianty quantification, allowing us to:

1. Considerably **reduce the number of simulator calls** compared with the nested approach (in the proposed example was 6 orders of magnitud lower).
2. Obtain **multiple solutions** to the MORO problem with different levels of robustness without additional simulator calls.
3. **Re-use the metamodels** in new optimization processes or computationally demanding applications.

Future works:

1. High-dimensional structural problems.

Thanks for your attention!

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