Robust Optimal Design of structures via Kriging models

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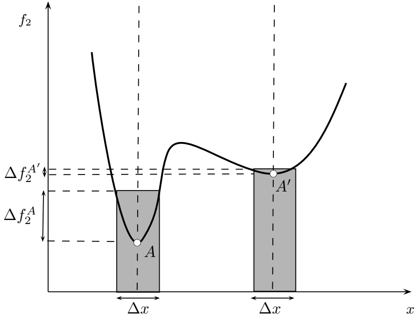
Outline

Robust Optimal Design
Motivation

The optimization under uncertainty aims to obtain optimal designs less sensitive to the uncertainties inherent to the structural parameters.

Sources of uncertainty:

- Applied loads
- Spatial positions of joints
- Section properties
- Material properties
- Environmental conditions.



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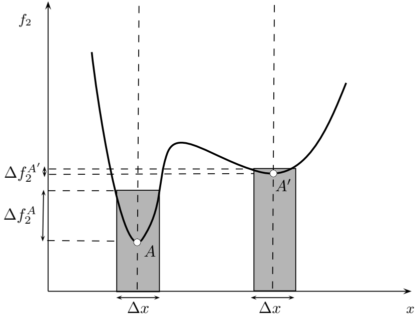

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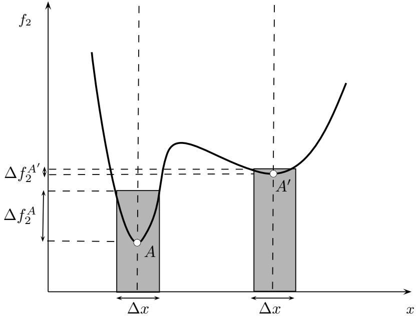

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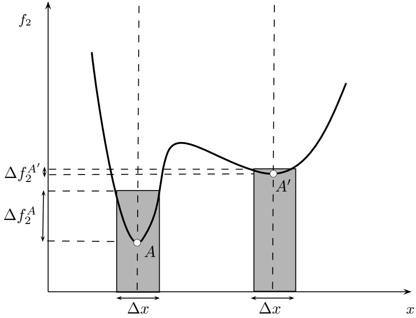

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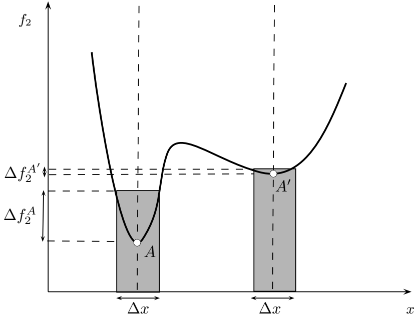


Robust Optimal Design
Formulation

Deterministic Optimal Design (DOD)

$$\begin{aligned} \min_{\mathbf{x}\in\mathbb{R}^n} \ & f(\mathbf{x}) \\ s.t. \ & g_i(\mathbf{x}) \leq 0 \quad i = 1, \cdots, m_i \\ & h_j(\mathbf{x}) = 0 \quad j = 1, \cdots, m_j \\ & \mathbf{x}^{\text{lower}} \leq \mathbf{x} \leq \mathbf{x}^{\text{upper}} \end{aligned} \tag{1}$$

Robust Optimal Design (ROD)

$$\begin{aligned} \min_{\mathbf{x}\in\mathbb{R}^n} \ & \{\mu_{f(\mathbf{x},\mathbf{z})}(\mathbf{x}), \sigma_{f(\mathbf{x},\mathbf{z})}(\mathbf{x})\} \\ s.t. \ & \mu_{g_i(\mathbf{x},\mathbf{z})}(\mathbf{x}) + \beta_i \sigma_{g_i(\mathbf{x},\mathbf{z})}(\mathbf{x}) \leq 0 \quad i = 1, \cdots, m_i \\ & \sigma_{h_j(\mathbf{x},\mathbf{z})}(\mathbf{x}) \leq \sigma_j^+ \quad j = 1, \cdots, m_j \\ & \mathbf{x}^{\text{lower}} \leq \mathbf{x} \leq \mathbf{x}^{\text{upper}} \end{aligned} \tag{2}$$

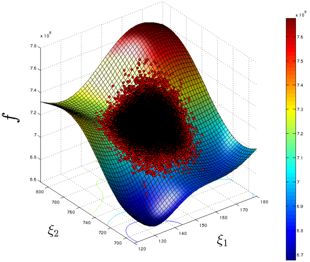
$\mathbf{x} \equiv$ vector of design variables, $\mathbf{z} \equiv$ vector of random parameters

Optimization under uncertainty
Uncertinty quantification (UQ)

The *kth* statistical moments of the structural performance can be analytically expressed using a multi-dimensional integral (3).

$$E\{g^k(\mathbf{x})\} = \int_{\Omega} g^k(\mathbf{x}) \cdot f_{\mathbf{X}}(\mathbf{x}) \cdot d\mathbf{x}, \tag{3}$$

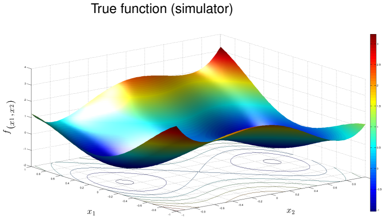
The main challenge is how to solve the multidimensional integration.

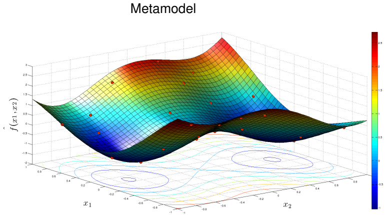


UQ
(Monte-Carlo)

Meta-modelling

Consist of replacing a computationally expensive simulation model by a mathematical approximation which is much faster to evaluate.





1. To sample the function to be predicted.
2. To create a mathematical approximation using statistical considerations.
3. To evaluate the accuracy of the mathematical model.

Kriging models

Kriging model

Kriging models assume that the simulator can be approximated by a sample path of a Gaussian stochastic process $\mathscr{G}(\mathbf{x})$

Prior mean

$$E[\mathscr{G}(\mathbf{x})] = \boldsymbol{f}(\mathbf{x})^{\mathrm{T}}\ \boldsymbol{\beta}$$

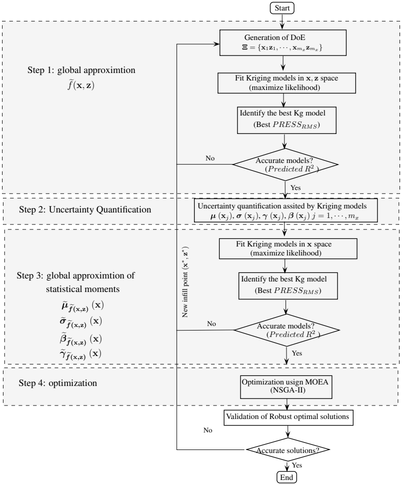
Prior covariance function

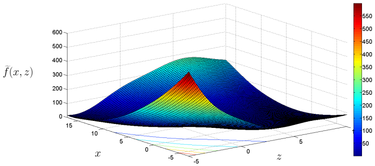
$$Cov[\mathscr{G}(\mathbf{x}), \mathscr{G}(\mathbf{x}')] = \alpha^2 \exp\left(\sum_{\mathrm{i}=1}^{\mathrm{n}} -\frac{|\mathrm{x_i} - \mathrm{x'_i}|^s}{\phi_{\mathrm{i}}}\right)$$

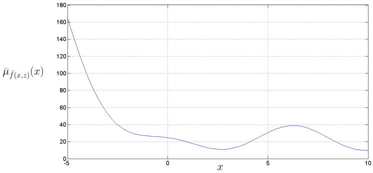
The parameters $\boldsymbol{\beta}$, α^2 and ϕ are unknown a priori and are determined from the set of simulator responses

$$\hat{y}(\mathbf{x}) \equiv \mathrm{E}[\mathscr{G}(\mathbf{x})|\mathscr{Y}] = \boldsymbol{f}(\mathbf{x})^{\mathrm{T}}\hat{\boldsymbol{\beta}} + \mathbf{r}(\mathbf{x})^{\mathrm{T}}\mathbf{R}^{-1}(\mathscr{Y}^{\mathrm{T}} - \mathbf{F}\hat{\boldsymbol{\beta}})$$

Proposed approach
Flowchart







Problem definition
four-bar truss structure (Dolsitis et al., 2004)

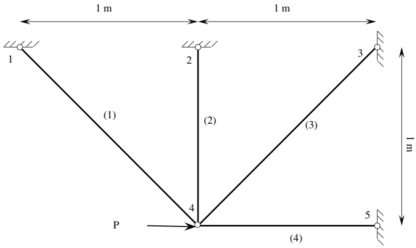
$$\min_{A_1,A_2} \{\widetilde{\mu}_{\widetilde{u}(A_1,A_2,E_1,E_2)}(A_1,A_2), \widetilde{\sigma}_{\widetilde{u}(A_1,A_2,E_1,E_2)}(A_1,A_2)\}$$
$$s.t.\ w \leq 5$$
$$0 \leq A_{1,2} \leq 2$$

Design variables:

- A_1 (bars 1 and 3).
- A_2 (bars 2 and 4).

Random parameters:

- $E_1 \sim \mathcal{N}(210, 21)$ (bars 1 and 3).
- $E_2 \sim \mathcal{N}(100, 15)$ (bars 2 and 4).



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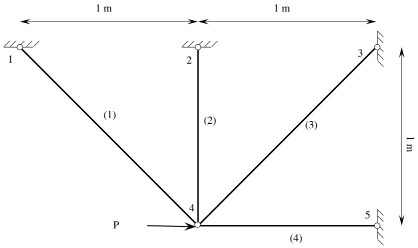
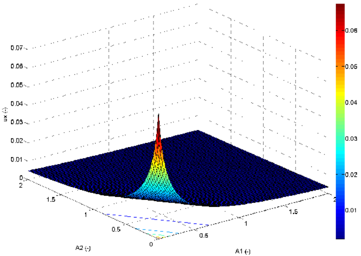


Global approximation deflection

The Kriging model was created based on 100 pieces of information obtained from an Optimized Latin Hipercube Sampling, wich was improved using 100 aditional infill samples.

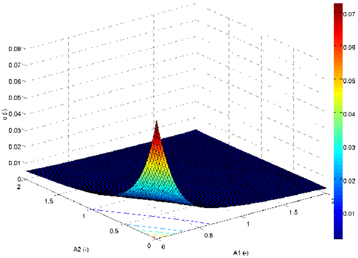
True function (simulator)

$u(A_1, A_2, E_1, E_2)$



Kriging model

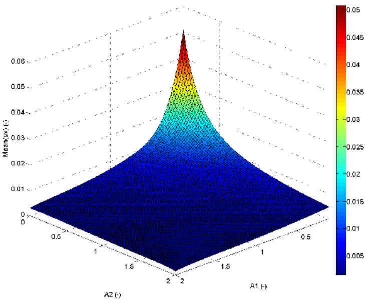
$\widetilde{u}(A_1, A_2, E_1, E_2)$



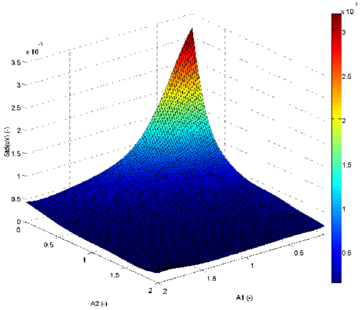
Stochastic meta-models deflection

- The statistical moments were obtained by Monte Carlo simulation for a sample size of 10000.

$$\widetilde{\mu}_{\widetilde{u}(A_1, A_2, E_1, E_2)}(A_1, A_2)$$

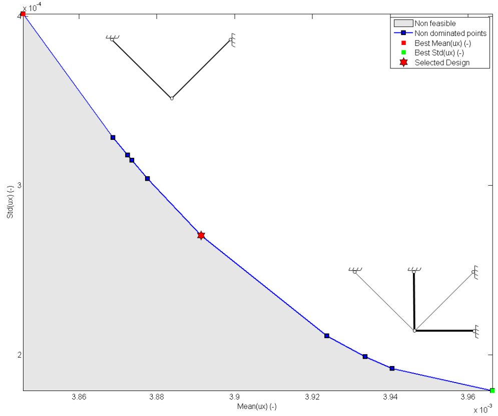


$$\widetilde{\sigma}_{\widetilde{u}(A_1, A_2, E_1, E_2)}(A_1, A_2)$$

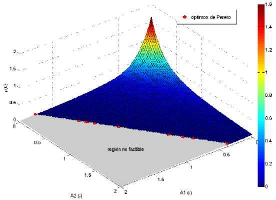


Multi-objective optimization
Robust Pareto fronts & feasible region

Robust Pareto frontier



$\widetilde{\mu}_{\widetilde{u}(A_1,A_2,E_1,E_2)}(A_1, A_2)$



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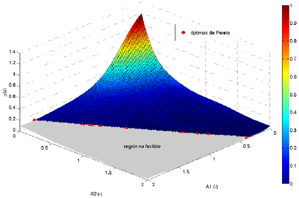
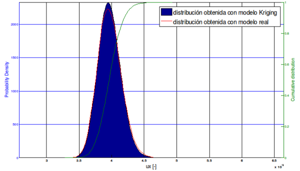


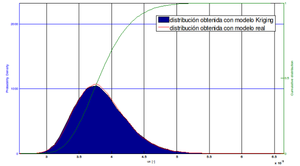
Multi-objective optimization
Accuracy

$$min\ \sigma_u(\mathbf{x})$$



	Kriging	Real	Error(%)
$\mu_u(\mathbf{x})$	3.965E-3	3.967E-3	0.033
$\sigma_u(\mathbf{x})$	1.721E-4	1.786E-4	3.629

$$min\ \mu_u(\mathbf{x})$$



	Kriging	Real	Error(%)
$\mu_u(\mathbf{x})$	3.843E-3	3.846E-3	0.073
$\sigma_u(\mathbf{x})$	4.007E-4	3.961E-4	1.151

Concluding remarks

Conclusions:

1. The proposed approach has shown to be **efficient** in low-dimensional problems that involved computationally demanding simulation models.
2. The global approximation allow us not only to **speed up** the optimization algorithm, but also to **explore** the design space, improving the formulation of the problem.
3. The Kriging models can be **re-used** in new optimization processes or computationally demanding applications.

Future works:

1. High-dimensional structural problems.

Thanks for your attention!

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