Robust Topology Optimization of Structures using Kriging Models

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• Introduction.
• Robust topology optimization.
  • Formulation.
  • Algorithm for robust topology optimization.
• SIMP method.
• Uncertainty.
  • Quantification.
  • Propagation.
• Kriging Models.
• Examples.
• Conclusions.
• Structures used in real world should consider the effect of an uncertainty environment.

• Design under uncertainty:
  • Reliability-Based Design Optimization (RBDO). Minimum failure probability.
  • Robust Design Optimization (RDO). Solution insensitive to uncertainties.

• Robust Topology Optimization (RTO), is a combination between Robust Design Optimization (RDO) and Topology Optimization (TO).

• Some works about RTO under uncertainty in loading:
  • Chen et al. (2010).
  • Dunning and Kim (2011; 2013).
  • Zhao and Wang (2014-a; 2014-b).
Robust topology optimization: Formulation

\[ \min. \quad C(u) \]

Subject to:
\[ K(\rho)u(\rho) = f \]
\[ V \leq V_{\text{max}} \]
\[ 0 \leq \rho \leq 1 \]

\[ \min. \quad E[C(u, z)] \]

Subject to:
\[ K(\rho, z)u(\rho, z) = f(z) \]
\[ V(z) \leq V_{\text{max}} \]
\[ 0 \leq \rho \leq 1 \]

\( C(\cdot) \): compliance,
\( u \): displacement field,
\( K \): stiffness matrix,
\( f \): load vector,
\( \rho \): densities vector,
\( z \): uncertainty variables,
\( E[\cdot] \): expected value.
1 Deterministic.
2 Robust (Montecarlo).
3 Robust (Kriging + Montecarlo).
• Density based method (Bendsøe 1989; Rozvany et al. 1992)

\[ 0 \leq \rho_e \leq 1. \]

• Penalization for intermediate densities:

\[ E_e(\rho_e) = E_{\text{min}} + (E_0 - E_{\text{min}})\rho_e^p. \]

\[ E(x) \text{ Young modulus updated,} \]

\[ E_0 \text{ Young modulus solid material,} \]

\[ E_{\text{min}} \text{ Young modulus for void material,} \]

\[ p \text{: penalization factor.} \]

• Density filter is used to avoid mesh dependent solution and checkerboard patterns.

• The topology optimization problem is solved by means of a standard optimality criteria method (Sigmund 2001).
• In this work are considered independent random variables in loading.
  • Magnitude, Direction, Position.
• A random variable is defined by a probability density function (pdf).

<table>
<thead>
<tr>
<th>pdf</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beta</td>
<td>a: first shape parameter, b: second shape parameter</td>
</tr>
<tr>
<td>Chisquare</td>
<td>v: degrees of freedom</td>
</tr>
<tr>
<td>Exponential</td>
<td>μ: mean</td>
</tr>
<tr>
<td>Gamma</td>
<td>a: shape parameter, b: scale parameter</td>
</tr>
<tr>
<td>Inverse Gaussian</td>
<td>μ: scale parameter, λ: shape parameter</td>
</tr>
<tr>
<td>Longnormal</td>
<td>μ: log mean, σ: log standard deviation</td>
</tr>
<tr>
<td>Normal</td>
<td>μ: mean, σ: standard deviation</td>
</tr>
<tr>
<td>Poisson</td>
<td>λ: mean</td>
</tr>
<tr>
<td>Uniform</td>
<td>a: lower endpoint, b: upper endpoint</td>
</tr>
<tr>
<td>Weibull</td>
<td>a: scale parameter, b: shape parameter</td>
</tr>
</tbody>
</table>
• Performance expected value $\mu_f$ is defined as

$$\mu_f = E[C(u, z)] = \int C(u, z) \, \text{pdf}(z) \, dz$$

• This integral can be difficult to evaluate.

• Therefore approximate methods are used:
  • Simulation methods: Montecarlo (MC), Quasi-MC, Latin Hypercube,…
  • Expansion methods: collocation method, perturbation method,…
  • FORM, SORM.
  • Meta-models: response surface method, Kriging methods,…
  • Approximate integration: Univariate Dimension Reduction (UDR), Bivariate Dimension Reduction (BDR), …

• The Montecarlo (MC) method is used in this work.
Monte Carlo Methods

\[ \mu_f = E[C(\rho, z)] \approx \frac{1}{N_{MC}} \sum_{i=1}^{N_{MC}} C(\rho, z_i) \]

- The accuracy of estimate is good if \( N_{MC} \) is large (>10000).
- The computational cost is proportional to \( N_{MC} \).
• Kriging Models are interpolation models.
• They are used to surrogate a true response $f(x)$.
• Reduce the computational cost

\[
\hat{f}_l(x) = F(\beta_{il}, x) + Z_l(x) \quad l = 1, \ldots, q
\]

Regression model
\[
F(\beta_{il}, x) = \sum_{i=1}^{p} \beta_{il} f_i(x)
\]

Stochastic process
\[
E[z_i(w), z_l(x)] = \sigma^2 \mathcal{R}(\theta, w, x), \\
E[z_i(x)] = 0, \\
l = 1, \ldots, q
\]
• Design steps:

1. Latin Hypercube Design (size $N_K$)
   \[ S = [s_1 \ s_2 \ldots \ s_{N_K}] \).

2. Evaluate structural response (displacement)
   \[ u = [u_1 \ u_2 \ldots \ u_{N_K}] \).

3. Generate the Kriging Model
   \[ \hat{u}(x) \approx f(S, u) \).

![Diagram](image-url)
• Elasticity module:
  • $E_o/E_{\text{min}}$: $1/10^{-4}$,
  • Poisson’s ratio: 0,3.

• Finite elements:
  • 4-node bilinear,
  • Unit sized elements.

• SIMP parameters:
  • Penalization factor: $p = 3$,
  • Filter radio: $r = 2$.

• Kriging Model
  • Regression function: first order polynomial.
  • Correlation function: exponential.
Example 1: Cantilever beam (1)

\[ \frac{V_f}{V_o} = 0.3 \]

- \( f \): Punctual load.
- \( F \): Magnitude.
- \( \theta \): Direction.
- \( S_x \): Load position on x axis (0-1).
- \( S_y \): Load position on y axis (0-1).

Case 1: Uncertain load position.
Case 2: Uncertain load magnitude.
Case 3: Uncertain load direction.
Example 1: Cantilever beam (2) Uncertain load position

Montecarlo:
E[C] = 34.83

Montecarlo-Kriging:
E[C] = 35.27 (+1.2 %)

Case 1:

\[ \frac{V_f}{V_o} = 0.3; \]
\[ F : 1; \]
\[ \theta : +90^\circ; \]
\[ S_x : 1.0, S_y : \text{Normal (0.5, 0.17)}; \]
\[ Nel_x : 30; Nel_y : 30; \]
\[ N_{MC} : 10000; N_K : 10. \]
**Example 1:**
Cantilever beam (3) Uncertain load magnitude

**Case 2:**

\[ \frac{V_f}{V_o} = 0.3; \]

\[ F : \text{Normal} (1, 0.033); \]

\[ \theta : +90^\circ; \]

\[ S_x : 1.0, S_y : 0.5; \]

\[ Nel_x : 30; Nel_y : 30; \]

\[ N_{MC} : 10000; N_k : 6. \]

Montecarlo:

\[ E[C] = 26.65 \]

Montecarlo-Kriging:

\[ E[C] = 26.60 (-0.2 \%) \]
Example 1: Cantilever beam (4) Uncertain load direction

Montecarlo: $E[C] = 6.04$

Montecarlo-Kriging: $E[C] = 6.05 \text{ (-0.2 %)}$

Case 3:

$V_f/V_o = 0.3$; $F = 1$;
$\theta : \text{Normal (0\degree \ 5\degree)}$;
$S_x : 1.0, S_y : 0.5$;
$Nel_x : 30; Nel_y : 30$;
$N_{MC} : 10000; N_K : 6$. 

Montecarlo - Kriging: $E[C] = 6.05$ (-0.2 %)
Example 2: Inverted T (1)

\begin{align*}
N_{MC} & = 10000, \\
N_K & = 6, \\
V_f/V_o & = 0.5.
\end{align*}

Uncertain loads \( F_1, F_2, \)
Normal \( (\mu_F = 5.0, \sigma_F = 0.5), \)
Normal \( (\mu_\theta = -90^\circ, \sigma_\theta = 14.3^\circ). \)

12672 bilinear elements, 
2570 dof.
Example 2: Inverted T (2)

Deterministic design $C = 2786.91$

Robust design (MC), $E[C] = 3341.70$. 
$t_i = 99388 \text{ s}$

Robust design (MCK), $E[C] = 3328.20 \ (-0.40 \%)$. 
$t_i = 44 \text{ s} \ (-99.95 \%)$
Example 2: Inverted T (3)

Deterministic design

\[ C = 2786.91 \]

Robust design with uncertain load

\[ E[C] = 3376.82 \quad \text{SD}[C] = 701.74 \]

Deterministic design with uncertainty load

\[ E[C] = 7657.80 \quad \text{SD}[C] = 6519.0 \]

E[·] : expected value
SD[·] : standard deviation
• A general methodology for topology optimization with uncertainty is presented in this work.

• Loading uncertainties are considered in magnitude, direction and position, like independent random variables.

• Montecarlo method is used to propagate the uncertainty to the response and a Kriging Model is used to reduce the computational cost.

• The proposed methodology (MCK) is accurate and very efficient. The computational cost is much lower than standard Montecarlo method.
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Thanks for your attention
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